AN
INTRODUCTORY TEXT-BOOK
OF
INDUCTIVE LOGIC

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FOR THE USE OF JUNIOR STUDENTS

BY

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"A TEXT-BOOK OF DEDUCTIVE LOGIC."

CALCUTTA:

S. K. LAHIRI & Co.,

56, COLLEGE STREET,

1916.

Printed at the Baptist Mission Press.
41, Lower Circular Road, Calcutta.

PREFACE

At the request of many Professors who teach the subject to the Intermediate classes in our Colleges, I have written this Introductory Text-book of Inductive Logic. I have tried to exclude from it all that would be unintelligible to those who begin to study the subject without any special scientific knowledge. The book presupposes only an elementary knowledge of Deductive Logic and an acquaintance with the phenomena of Nature, which a schoolboy acquires or may acquire in the ordinary course of his life. My object throughout has been to produce a book which would promote the study of a subject which has been rightly regarded as an Introduction alike to Science and Philosophy. I trust that the study of this book will be a good preparation for a student, for his subsequent course, whether it be Philosophy or Science.

The Introductory Text-book is completed in sixteen chapters specially designed for junior students. The Appendix contains additional matter for advanced students. In the Note (pp. 93–96) I have referred to certain points which should not, I thought, be entirely overlooked even in an Introductory Text-book.

My best thanks are due to Mr. K. C. Bhattacharjee, M.A., P.R.S., Professor of Logic and Philosophy in the Bethune College, Calcutta, who has made many valuable suggestions in the course of perusing the manuscript and correcting the proof-sheets. I should be obliged if the Professors who may use this book, would communicate to me their suggestions for its improvement.

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7, BALLYGUNGE CIRCULAR ROAD,
CALCUTTA,
April, 1916.

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INTRODUCTORY TEXT-BOOK OF INDUCTIVE LOGIC.

CHAPTER I.

MILL'S VIEW OF INDUCTIVE LOGIC.

§ 1. Mill divides all truths, according to the way in which they are known, into (*i*) Intuitive and (*ii*) Inferential.

(*i*) Intuitive truths are known immediately without any reasoning. Mill regards the facts of consciousness—the mental feelings and bodily sensations—as known intuitively. "I am hungry now," "I was vexed yesterday," "I am touching this paper and seeing its whiteness," etc., are propositions which are known immediately without reasoning. They express certain facts of consciousness which are directly known and which are not inferred from other facts. According to Mill all such facts are particulars existing at a certain time.

(*ii*) Inferential truths are known mediately by reasoning. They are inferred from intuitive truths by the processes of Naming, Definition, Classification, Generalisation, etc., which form the subject-matter of Logic. From the particular facts known by Intuition, a general proposition may be inferred according to the Canons laid down by the Logic of Induction. According to Mill we usually reason from some particular facts to some others with which they are known by experience to be

connected. The universal type of reasoning, according to him, is that if a particular or individual thing, *A*, resembles another particular or individual thing, *B*, then what is true of *A* will be true of *B*, the certainty of the inference depending on the degree of resemblance between *A* and *B*. What is true of *A* and *B* will be true of any other particular or individual thing which resembles *A* and *B* in the same attribute or attributes in which they resemble each other. *A*, *B*, *C*, etc. are particulars resembling one another, say in the attribute *m*; then if the attribute *n* is known by experience to be connected with *m* and true of the particular *A*, *n* will be true of the particular *B* or *C*, or any other possessing the attribute *m*. *A*, *B*, *C*, etc. will thus form a class and give rise to a general proposition affirming the connection of the attribute *m* with *n*, that is, any particular or individual thing possessing the attribute *m* will also have the additional attribute *n*. If we call this class *S* with *m* as its common attribute or connotation, we may state the general proposition in the usual form, "All *S* is *P*" where '*P*' is an adjective signifying the attribute *n*. For example, in the general proposition "All men are mortal," 'humanity' and 'mortality' correspond respectively to *m* and *n*, the particular things are individual men having the attribute, humanity, in common, with which the attribute, mortality, is known by experience to be connected. The process by which the universal proposition, "All men are mortal," is established from the particular cases of the death of individual men is called Induction and the operation by which it is applied to any living man or men is called Deduction by Mill.

§ 2. According to Mill, all universal propositions which are not merely verbal (analytical), or deduced from verbal propositions, are generalisations from particulars known by experience. It is the business of Inductive Logic to lay down the conditions to which we must

conform in order that a general proposition inferred from one or more particulars, may be true

Mill holds that the Laws of Thought, the Axioms of Geometry etc., which are usually regarded as self-evident and intuitive, are also generalisations from experience. According to him, intuition gives us no knowledge of any universal principle. It only reveals to us the particular facts of consciousness. In our ordinary perceptions and observations there are two elements, (1) Intuitive, or the elements of original sensations or pure perceptions of the several senses, and (2) Inferential, or the elements added to them by reasoning. In order that our perceptions and observations may be true, the reasoning involved in them must conform to the conditions of correct reasoning. It is evident, therefore, that Mill is bound to analyse our ordinary perceptions and observations and distinguish the inferential elements in them from those that are primary and intuitive. Our ordinary perceptions and observations are, he points out, the results of reasoning, and should be distinguished from the pure sense-perceptions which are intuitive. As the end of Logic, according to Mill, is the attainment of real or scientific truth, it is necessary that the data supplied by observations should be true and that the inferential elements contained in the observations should conform to the laws of Logic. He holds that there are no rules of Logic regulating pure perceptions or pure observations, that is, the intuitive elements involved in our ordinary perceptions and observations. They form the ultimate data of all reasoning. They depend on our physical and mental constitution and on the environment. Logic assumes them as they are in normal human beings.

No syllogistic reasoning is possible unless there are general or universal propositions. In the first figure to which all syllogisms may be reduced, the major premise must be universal. Mill holds that as all universal propo-

sitions are generalisations from experience, no valid (true) syllogistic reasoning is possible unless we have valid generalisations. We may assume them or they may be analytical, but the conclusions will then be hypothetical and not true. In order that the conclusion of a syllogism may be true, the premisses must be true; and of these one, the major, being universal must be a generalisation from particulars known by experience. The problem for Mill's Logic is, therefore, to lay down the conditions to which a generalisation must conform in order that it may be true.

§ 3. Mill finds a solution of this problem in what he calls the Principle of the Uniformity of Nature. This principle is itself a generalisation from experience. By "Nature" he means the whole of phenomena, physical or mental, which are known or capable of being known by us. By Uniformity he means that there are constant relations or connections among these phenomena. He points out that the uniformity of nature consists of many uniformities in different departments of Nature. He classifies them under the three heads of (1) Coexistence, (2) Succession, and (3) Likeness. The phenomena of Nature coexist in space, succeed each other in time, and resemble one another in certain qualities, according to laws which are investigated in different sciences. Under Succession he recognises a special kind of uniformity, which he distinguishes from others and generalises under the Law of Causation. This Law means, according to Mill, (1) that every phenomenon has a cause, and (2) that the cause of a phenomenon is a phenomenon or an aggregate of phenomena, which invariably and unconditionally produces the phenomenon. Mill shows that the Law of Causation and the Principle of Uniformity of Nature form the basis of all scientific investigations. The observations and experiments in science are conducted on the assumption of their truth. The generalisations in Physics and Chemistry and other

Sciences are so many different instances of the two fundamental principles of Causation and Uniformity of Nature.

§ 4. According to Mill, an Induction is a valid generalisation from some to all particulars of a class. By a "particular" he means an individual belonging to the class. "Socrates, Plato, Aristotle and others have died; therefore all men will die" is an Induction Is it valid? It is valid if it conforms to the principle of the Uniformity of Nature—if a constant connection can be shown to exist between the attribute or attributes common to Socrates, Plato, Aristotle and others, i.e. to men who have died, and the attribute of dying or death. What is the attribute in them which is so connected with death? They all have an organised body; and it must be shown by Inductive Methods that there is a constant connection between organisation and death—that wherever an individual has an organised body, it has in it also the elements leading to the dissolution of the organisation (connection of Coexistence) or that the phenomena constituting an organised body will be invariably followed by the phenomena of its dissolution (connection of Succession or Causation). The induction "All men will die" or "All men are mortal" may be established by the Method of Simple Enumeration. It may be shown that all individual men in past ages died, that many men in the present age have died, that there are no cases of men living beyond a certain age, that, so far as our experience goes, all men die after a certain age and that therefore all men who are now living and who may be hereafter born will die, i e., all men are mortal. The Method of Simple Enumeration is the most popular method of induction as it is also the most ancient. "Food nourishes," "Water quenches thirst," "Fire burns," "Liquids expand by heat," "Bodies fall to the earth," "Crows are black," "Swans are white," "Plants have flowers," "Animals have power of locomo-

tion," "Metals conduct heat," etc., are inductions established by this method. Mill recognises it but points out that as it cannot establish the relation of cause and effect, the method is not of much use for scientific purposes. For a valid scientific induction it is necessary to show that the subject and the predicate are causally connected—that the attribute connoted by the subject, or the phenomenon signified by it, is invariably and unconditionally followed by the attribute connoted by the predicate, or the phenomenon signified by it. In the inductions given above such a connection has not been shown to exist. In fact we are not sure that some of them are valid inductions. They are no doubt generalisations from our experience. But a larger experience may or may not confirm them. As a matter of fact some of them are not universally true. There are black swans; there are plants that have no flowers; there are animals without the power of locomotion. In the case of the others it is necessary to state definitely the conditions under which they are true. Mill, therefore, distrusts the Method of Simple Enumeration for scientific purposes and formulates certain methods which he calls the "Methods of Experimental Inquiry" and which are known as Mill's "Methods of Induction."

The Induction "All men are mortal" may be established by showing that there is a causal relation between the phenomena of human life and those of death. For this purpose it would be necessary to analyse both the groups of phenomena into their constituent elements. The phenomena of human life include those of animal life; and the latter those of an organised living body; and it may be shown that the phenomena of organisation are invariably followed by those of the dissolution of the organism. An organism consists of certain organs performing the function of life. It may be shown that the organs are so constituted and the natural environment is such that after a certain period they invariably cease

to work, and thus bring about the death of the organism; that is, there is a causal connection between the phenomena constituting the life of an organism and those constituting its death. If such a connection can be established, the induction will be a valid one, according to Mill. The other popular inductions given above will be valid if a causal connection can be established between the subject and the predicate of the propositions, that is, between the phenomena signified by them. Mill's Experimental Methods of Inquiry are methods for discovering and proving such connections among the phenomena of Nature.

CHAPTER II.

PERCEPTION AND OBSERVATION.

§ 1. Ordinary observations are inferences from direct perceptions

Inductive Logic is concerned with the truth of phenomena known by inference, as distinguished from what is known by intuition. The ultimate data of all inference are facts known by intuition. There is great difference among philosophers as to what is exactly known by intuition. We have seen that, according to Mill, intuition gives us a knowledge only of our mental feelings and bodily sensations. A sensation is a mental state produced by an external object. A perception is the reference of the sensation to the external object and implies, according to him, an act of inference According to others, a perception gives us a direct and immediate knowledge of an external object. The word perception is sometimes used in a wider sense, including our immediate knowledge of the internal subject. The latter is called Internal Perception as distinguished from the former which is called External Perception. Perceptions are also distinguished as original and immediate from those which are acquired and mediate. The former are intuitive while the latter are inferential. In our ordinary perceptions and observations, there is a combination of both the elements.

"What we are said to observe," says Mill, "is usually a compound result of which one-tenth may be observation, and the remaining nine-tenths inference."—Bk IV, Chap I, section 2. By 'observation' he means here an immediate perception as distinguished from an acquired

or mediate perception in which there are elements of inference. "In almost every act of our perceiving faculties, observation and inference are intimately blended" "I affirm, for example," says Mill, "that I hear a man's voice. This would pass, in common language, for a direct perception. All, however, which is really perception, is that I hear a sound. That the sound is a voice, and that voice the voice of a man, are not perceptions but inferences. I affirm, again, that I saw my brother at a certain hour this morning. If any proposition concerning a matter of fact would commonly be said to be known by the direct testimony of the senses, this surely would be so. The truth, however, is far otherwise. I only saw a certain coloured surface; or rather I had the kind of visual sensations which are usually produced by a coloured surface; and from these as marks, known to be such by previous experience, I concluded that I saw my brother. I might have had sensations precisely similar, when my brother was not there. I might have seen some other person so nearly resembling him in appearance, as, at the distance, and with the degree of attention which I bestowed, to be mistaken for him. I might have been asleep, and have dreamed that I saw him; or in a state of nervous disorder, which brought his image before me in a waking hallucination. In all these modes, many have been led to believe that they saw persons well known to them, who were dead or far distant. If any of these suppositions had been true, the affirmation that I saw my brother would have been erroneous; but whatever was matter of direct perception, namely, the visual sensations, would have been real. The inference only would have been ill-grounded. I should have ascribed those sensations to a wrong cause."

§ 2. Immediate Perceptions.

Each sense has its immediate or direct perceptions. Sight, for instance, gives us a knowledge of the colours

of objects; Hearing of the sounds produced by them; Active Touch, of the resistances offered by them and of their forms and sizes; Smelling of their odours; Tasting, of their flavours. The colour, for example, of an orange is known by Sight, its form, size and solidity by Active Touch, its scent by Smelling, its flavour by Tasting. These are direct or immediate perceptions of an orange by the several senses. There is much difference of opinion among psychologists as to the exact knowledge supplied by each sense. All would, I think, agree that the original sensations of each sense constitute its direct perceptions of an object and that the qualities of the object are founded on these sense-perceptions. They would differ in their account of what is implied by each sense-perception. Is an object implied by every sense-perception? If so, what is its nature? Is it implied by some sense-perceptions only? If so, what are they? Does the perception of colour and sound imply any perception of extension? How is an object as a source of resistance and as occupying space known? Is it an inference or an intuition? A thorough discussion of these questions belongs to Psychology and Metaphysics.

§ 3. Mediate Perceptions.

A mediate perception of an object is an inference from an immediate perception of it. The inference of the taste of an orange from its appearance is a mediate perception. The inference of the distance of an object from its direct visual perceptions is a mediate perception acquired by experience. So are also the perceptions of the form and size of an object at a distance from the sensations of sight or rather from the perceptions of its visual figure and size. The immediate perceptions of one sense become associated by experience with the immediate perceptions of another sense; and the one set may be inferred from the other. The inferences will be true or false according as they do or do not conform to the rules of Logic.

§ 4. Internal Perception.

In Internal Perception there is no special organ of sense; but the mind directly perceives its feelings, its desires, its ideas, its activities, etc. It is sometimes called Introspection, i.e., looking within. It is also called Self-consciousness, i.e., the consciousness of the self as distinguished from the not-self. Self-consciousness is direct and immediate knowledge of the phenomena of one's own mind. The mental phenomena are so connected with one another that it is possible to draw inferences as in the case of the phenomena of the external world. From the immediate consciousness of a particular feeling may be inferred the desire that is likely to follow. From the immediate consciousness of an idea may be inferred the feeling that accompanies or is associated with it and the volition that is likely to follow. From a knowledge of the connections of the phenomena in one's own mind, inferences may be drawn in regard to the phenomena of another person's mind; and these may be verified by his gestures and actions or by direct reference to his consciousness.

As in the case of External Perception, Philosophers differ in their account of Internal Perception. The chief point of difference in the latter case is as regards the nature of the Self or the Mind. Does it merely consist of the phenomena, or is there a unity or a permanent reality which is the ground of the phenomena and which is directly known along with them?

§ 5. The word perception has a still wider import than has been noticed above. It means also a direct knowledge of the relation between two given things. For example, the consciousness of the likeness or unlikeness of two things, say, in regard to their colour, is also an immediate perception. Likewise, the equality or inequality of two lines placed side by side is known by direct perception, i.e., intuitively. The identity or difference of two things in respect to any particular attribute is

known by immediate perception. The relation of co-existence or of succession between two given phenomena is perceived directly without any reasoning. It is held by some philosophers that the knowledge of the relation of A to C from the knowledge of the relation of each to B is also direct and immediate knowledge, i.e. an intuition, as in the case of the first axiom: $A = B$; $C = B$, $\therefore A = C$. This is true if the three things remain unchanged, but will not be true in a world of flux. It is true of our ideas and concepts which are assumed to remain unaltered. In the case of phenomena, $A = C$ is an inference which must be verified by experience.

§ 6. The word perception as used in the phrases, Aesthetic Perception, Moral Perception, Spiritual Perception, etc., implies also direct and immediate knowledge of special kinds of facts and phenomena. The phenomena of beauty in the external and in the internal world are directly perceived by the mind. So are also the facts and phenomena of our moral and spiritual nature. Inductive Logic has to recognize the truth of all immediate and direct perceptions and lays down rules according to which true inferences may be drawn from them.

§ 7. The faculty of immediate perception, like all other faculties, grows by exercise. The child at first perceives all colours as more or less alike. But gradually he distinguishes them and shows his fondness for the bright colours. The colour-sense in an adult is more developed than in a child. Persons who are dealing in colours can easily distinguish the different shades of a colour, which, to an ordinary person, would appear to be the same. All this knowledge of the different colours and of their various shades is due to the growth of the faculty of visual perception. It is immediate and direct to the person in whom the faculty is developed. It is not the result of inference but of intuition. No amount of reasoning could produce in a person the knowledge of the difference between red and blue. This know-

ledge is intuitive and is only attained by a direct and immediate perception. The progress in intuitive knowledge caused by the growth of the faculty is true, not only of external perception, but also of the other kinds of perception noticed above. The facts directly known by the fully developed faculty of perception of every kind have to be recognised in Inductive Logic.

§ 8. Perception gives us immediate knowledge of what is present. Memory gives us immediate knowledge of what is past but has been perceived by us. The knowledge supplied by memory of past events and occurrences is as direct as the knowledge given by perception of what is happening now. It is true that memory sometimes becomes vague and faint and is not always reliable. It may require to be revived by circumstances; but, when thus revived, it recalls a past event with wonderful faithfulness. The facts recalled by memory must also be recognised by Inductive Logic.

CHAPTER III.

Observation and Experiment.

§ 1. Observation is the extension of immediate perception by inference. Experiment is the extension of observation by special arrangements of natural objects. By 'observation' we investigate the properties and laws of objects and phenomena as they occur in nature. By 'experiment,' objects are isolated from their natural combinations in nature, and phenomena are artificially produced for observation. Observation and experiment are necessary for obtaining the facts and phenomena of a science. The laws of the science are based upon these facts and phenomena. In Physics and Chemistry, for example, the general properties of material bodies, the special properties of liquids, solids and gases, the laws of expansion of these bodies by heat, the laws of chemical combination, the properties of the chemical elements and their various compounds, etc., have been determined by observation and experiment. In some sciences as Botany, Zoology, and Mineralogy, observation is the chief method of obtaining the facts, while in Chemistry and Physics, experiment is absolutely necessary for determining even the most elementary laws of these two sciences. It is by experiment that oxygen is prepared from its compounds in nature and its properties are determined. It is by experiment that the properties of the compounds which oxygen forms with the other elements such as carbon, nitrogen, etc., have been ascertained. The law of falling bodies has been determined by an experiment in which two bodies such as a piece of gold and a feather are allowed to fall through a glass cylinder which

has been emptied of its air. In such an experiment it is found that the two bodies fall to the bottom of the cylinder at the same time. The laws of reflection and refraction of light have been determined by experiments.

The truth of the laws of a science depends upon the truth of the facts and phenomena determined by observation and experiment. Observation and experiment must therefore be carefully conducted and the results accurately recorded for confirmation and future reference. For this purpose scientific apparatus as well as a complete system of terms are necessary—the former for producing and observing the phenomena and the latter for accurately recording them in language.

§ 2 Experiment has certain advantages over observation When a phenomenon is produced by an experiment, we know all the circumstances under which it is produced We can vary them as we like and notice the change in the phenomenon. We may thus determine a quantitative relation between the cause and the effect. But we can not produce a phenomenon by an experiment unless we know its cause. It is by observation that we can find out the probable cause of a given phenomenon. We observe many instances of it as it occurs in Nature and draw from them some inference as regards its probable cause. We may then experiment upon this cause and try to produce the phenomenon. The inference will be correct if the phenomenon is produced by it. But as it often happens the cause inferred may not produce the phenomenon. It may be only a part of the true cause and will not produce the phenomenon by itself Or it may consist of the cause and other circumstances which are irrelevant to the production of the phenomenon. For the elimination of these irrelevant circumstances experiment is necessary Take the phenomenon of vapour in the atmospheric air. What is its cause? We may observe many cases of the presence of vapour in the air on different days and in

different seasons. Its quantity increases in the rains, when the air becomes very humid. It is much less in the winter. It is more present in the air of Lower Bengal than in that of the Up-country. From these instances it is difficult to determine the conditions under which it is produced and the conditions under which its quantity in the air varies in different seasons and in different countries. It may be inferred that it is produced somehow from the water-sources of the country and that its quantity depends on the proximity to rivers and seas and on the prevalent winds, being greater, for example, during the monsoons. Its quantitative connection with the temperature of the air has been determined by experiment. It has also been determined by experiment that not only does water rise as vapour through heat as in the summer, but that all water-sources are giving off vapour throughout the year.

§ 3. In Inductive Logic it is usual to represent objects and their phenomena by symbols such as the letters of the alphabet. An object as consisting of a number of attributes, is represented by a combination of letters, each standing for an attribute. The attributes of an object are founded upon the phenomena of sense, which it produces in us. The attributes of a piece of chalk, for example, such as resistance (or solidity), figure, size, colour, etc. are grounded on the corresponding sense-perceptions. All the phenomena of objects are thus, on the one side, related to the mind and, on the other, to the object. They imply a sensibility of the former and a capacity of the latter to produce a sensation which is the content of the sense-perception. A piece of chalk is thus an aggregate of the various phenomena on which its attributes are grounded. The attributes are the capacities or potentialities of the chalk to produce certain sense-phenomena in us. It is these phenomena which form the subject-matter of science. The phenomena are represented by the same letters that stand for

the corresponding attributes. All phenomena may be traced to the attributes of an object or of a subject or mind The actions of one object upon another, or of one mind upon another, or of an object and a mind upon each other are phenomena implying attributes and may be represented by letters. A simple phenomenon is represented by a single letter and a complex or compound phenomenon by two or more letters. The terms "simple" and "compound" as applied to phenomena are relative. A piece of chalk is sometimes taken as one phenomenon and represented by a single letter and sometimes as a compound one represented by several letters, standing for the several sense-phenomena of resistance, figure, size, colour, etc. it produces in us.

In Inductive Logic the antecedents of a phenomenon or both the antecedents and the consequents in a phenomenon are represented by letters. In the latter case it is evident that the phenomenon is a very complex one, consisting of certain antecedents and certain consequent phenomena, each of which may again be complex. In the former case the phenomenon may be simple or complex. Even when we speak of the antecedent and the consequent in the singular, the phenomenon referred to may be complex.

CHAPTER IV.

CLASSIFICATION AND DEFINITION.

§ 1. By immediate perception, observation and experiment, we know individual things and phenomena; and the problem of classification is their grouping according to their resemblances and differences. They may be divided into two broad classes according as they are known by external or by internal perception. Those known by external perception occupy space and those known by internal perception occur in time and do not occupy space. All external things and phenomena thus form a very large class, having the attribute of being in space as their common quality, while all internal or mental things and phenomena form another very large class distinguished from the former by their not being in space. Both agree in being in time—things existing in time, and phenomena being changes in time.

All external things and phenomena may next be divided into classes according to the sense by which they are known. Thus there would be as many classes as there are senses giving us knowledge of external objects. Touch, sight and hearing are the most important knowledge-giving senses and there would thus be three classes, namely (1) Tangible, (2) Visible and (3) Audible. But these classes would not be exclusive of one another as one and the same individual thing may be an object of all the three senses. The classification would be one of attributes founded on conscious phenomena rather than of individual things. The latter may be classified according to the degree of resistance offered by them into (1) Solid, (2) Liquid, (3) Gaseous, (5) Ethereal. The first

three are known to have the element of resistance in different degrees, but the last—the medium of light, heat and electricity—appears to be without it or to have so little of it that the present scientific instruments have not been able to detect it.

All things offering resistance and occupying space may be next classified into those that have life and those that are without it, namely, (1) living (plants and animals) and (2) non-living (minerals, rocks, stones, chemical elements and compounds, etc.) Living things may be classified according as they have sensibility and power of locomotion or not, into animals and plants. Animals may be classified according to their organisation, i.e. the degree of life developed in them. So may also be plants. The classification of plants is the subject of Systematic Botany and that of animals of Zoology.

The first broad classification of plants is into (1) flowering, i.e. those that have flowers, and (2) non-flowering, i.e. those that are without any flowers, e.g. ferns, mosses, etc. The first broad classification of animals is into (1) Vertebrata, i.e. those that have the back-bone, and (2) Invertebrata, i.e. those without it, e.g. insects, molluscs, etc. One of the earliest classification of animals is according to their place of living—(1) land, (2) water, or (3) both (amphibious).

§ 2. The object of all these classifications is to bring together those things that resemble one another and to separate them from those from which they differ. The greater the number of qualities in which the individuals of a class resemble, and the more important these qualities, the more scientific is a classification. The importance of a quality depends on its persistency and its connection with other qualities in a group of individuals. The vertebral column is an important attribute for the classification of animals because it is present in all higher animals including fishes; and it is further found that it is correlated with a number of organs such as the skull,

the spinal cord, the heart, the brain, the organs of sense, etc. in an animal. Likewise the presence or absence of flowers is a very important attribute of plants. All flowering plants have many other common characters, while all flowerless plants are devoid of them, but have other common characters. A scientific classification takes into consideration all the attributes of the things to be classified and groups them according to their most important characters. A classification is said to be artificial if it is founded on one character for a particular purpose. If, however, the character is an important and persistent one, an artificial classification may pass into a scientific one. The classification of objects founded on the single quality of resistance and its varying degrees is a scientific one.

§ 3. Classification is a very important process in science. It gives rise to classes, each possessing certain common attributes. The common attributes constitute the connotation of the class-name. New individuals may be included in the class if they are found to possess the common attributes. The class metal arises from a classification of elements into (1) metals and (2) non-metals and is characterised by certain qualities. If a new element is discovered and is found to possess some of those qualities, it is inferred that it possesses also the others, and the element is referred to the class metal. The inference may be verified afterwards by experiment. Likewise if a new plant is discovered and found to possess some of the characters of a particular class, it is referred to the class under the belief founded on the Uniformity of Nature, that the other characters, for example, the fruit, will also be found in due course. The new plant may be identified by its flower; and the fruit characteristic of the class may be produced in proper season.

New attributes may be added to the common attribute or attributes of a class with the increased knowledge of

the things belonging to the class. With the progress of Physics and Chemistry there has been an addition to our knowledge of the common attributes of the class metal. Similar change has taken place in the connotation of the class-names, plant, animal, living body, etc. Our knowledge of the things denoted by them has increased with the progress of Biology, Botany and Zoology, and it has become necessary to re-model their classifications on new principles. Of these the most important principle is that of Evolution of living things from a few original forms. The old classifications of plants and animals are being transformed into genealogical trees.

§ 4. The preceding section shows the close connection between Classification and Definition. The definition of a class-name consists of the attributes common to the individual things belonging to the class. The definition will increase in content with the advance in our knowledge of the common attributes. Definitions of the names will change as the classification is adapted to our increasing knowledge of the things classified. The definitions of metals, plants, animals, living bodies, etc., have changed with our increased knowledge of the things. The more accurate and advanced is our knowledge of the things defined, the truer is the definition. In defining the term metals, the object at first is to find out its correct meaning in popular usage, then its meaning in scientific usage. We shall thus have a popular and a scientific definition of the term. But the philosophic problem of definition of metals goes deeper. It attempts to find out the most fundamental qualities which are inherent in the known metals, which distinguish them from other substances, and from which their other qualities are derived. This may not be the case with the present scientific definition of the term. A final and complete definition implies a final and complete knowledge of the individual substances denoted by the term. It is therefore an ideal at which Science aims in its investigation

of truth rather than an accomplished fact in regard to any class of natural objects.

§ 5. It is evident that classification presupposes (1) a system of names for describing the parts and actions of individual things; (2) a system of names signifying the qualities in which the individual things resemble one another; (3) a system of names for the classes which arise from the grouping of the individual things according to their resemblances and differences.

The first and the second have been called Terminology and the third Nomenclature.

In describing a plant, for example, names or words are required for its various parts and organs such as the roots, the trunk, the branches, the leaves, the flowers, the fruits, etc., and for noting the various modifications of these parts and organs. These names must have definite meanings and be sufficient in number to describe accurately and fully the various parts of an individual plant so as to be able to distinguish it from another. The parts and organs of one plant resemble those of another; and names are required to note these common qualities. These two systems of names constitute what is called Terminology. When the plants observed and described are grouped into classes according to their resemblances and differences, names are selected for these classes according to their position in a scheme of classification. These names, especially those standing for the lowest classes called infimæ species, constitute what is called Nomenclature. Botany, Zoology, Mineralogy, Chemistry have each its systems of Terminology and Nomenclature.

§ 6. Generalisation and Induction.

Generalisation is the process by which what is known to be true of one or more individuals of a class is inferred to be true of the whole class. For instance, it is observed that water and oil are solidified by cold and it is inferred that all liquids are solidified by cold. The general proposition which is the result of a generalisa-

tion is called Induction. Whether the Induction "all liquids are solidified by cold" is true or not will depend on further observation. If it cannot be verified by observation, recourse may be had to experiment. The liquid bodies that are not seen to solidify under the natural conditions of the cold weather may be solidified by cold produced artificially by properly devised experiments. The general proposition may be thus established by observation and experiment. It may be true of all the known liquids. But it may still be contradicted by some newly discovered liquid. In order that it may be established as a universal proposition, true of all liquids whatever, known or unknown, it is necessary to establish some connection between liquidity and cold on the one hand and solidity on the other, which will hold good under all conditions. Until such a connection is established, the generalisation must be regarded as empirical and may be falsified by a single instance to the contrary.

CHAPTER V.

THE DIFFERENT KINDS OF INDUCTION.

§ 1. Every valid induction is founded, according to Mill, on a constant relation or connection among the phenomena of Nature. He distinguishes three kinds of Induction corresponding to the relations of (1) Coexistence, (2) Likeness or Resemblance, (3) Succession.

(1) Inductions of Coexistence.

Some samples of gold have certain qualities; therefore, gold, wherever it may be found, will have the same qualities. This is an induction of coexistence. The qualities are connected with one another and coexist in every piece of gold. The connection among them is assumed to be a constant one on the ground of the Uniformity of Nature and is verified by experience. There are similar inductions founded on the coexistence of certain qualities in some samples of other chemical elements as also of chemical compounds and in some individuals of a class or species of plants or animals. To the class of Inductions of Coexistence belong all Laws of Nature which cannot be brought under the Law of Causation. The law that whatever has inertia gravitates is an Induction of Coexistence. No causal connection has been shown to exist between inertia and gravitating.

§ 2. (ii) Inductions of Likeness or Resemblance.

Every Induction of Coexistence is also an Induction of Likeness or Resemblance. The induction "all gold has certain qualities" may be stated in the form "all samples of gold resemble one another in those qualities." The induction "all material bodies gravitate" is the same as "all material bodies resemble one

another in the attributes of inertia and gravitating." The induction "all men are mortal" expresses the fact that all men resemble one another in the attributes, animality and mortality. Resemblance may vary in degree. In some cases it amounts to identity as in the samples of chemical elements and compounds. The different samples of oxygen, hydrogen, water, carbonic acid, silver, copper, all in their chemically pure state, have identical properties. What is true of one pure sample of a chemical element or compound is true of all pure samples of the same element or compound, because they resemble one another to such an extent that one sample cannot be distinguished from another. These inductions are founded on the uniformity of resemblance or identity between the different samples of the same substance. In other cases the resemblance is partial. It exists in the midst of difference. One man resembles another in certain qualities and differs from him in certain other qualities. What is therefore true of some men cannot be inferred of all men, unless what is inferred is causally connected with, or follows from, the qualities in which they resemble. For the quality inferred may be one which is connected with those qualities in which they differ from one another. It may be a special quality of a particular individual or of a particular race and may not be possessed by another individual or another race. Inductions founded on resemblance must therefore be carefully tested and brought under the Law of Causation.

§ 3. Inference from Analogy.

Inferences founded on the resemblance of two individuals to each other are distinguished as analogical. The earth resembles the planet Mars in certain qualities: it is therefore inferred by analogy that as the earth is inhabited by plants and animals, Mars is likewise inhabited by them. Such an inference will be only probable unless a causal connection can be established between the com-

mon qualities of the earth and Mars and the phenomena of plants and animals. Two brothers resemble each other in a number of qualities. What is true of one may be inferred of the other, if the quality inferred is connected with the qualities in which they resemble. The connection may be one of causation between the common qualities and the quality inferred; or the latter may follow from the former by way of deduction. Otherwise the inference will be only probable.

Analogical inferences which cannot be brought under the Law of Causation depend on the Laws of Coexistence and Resemblance. Two individuals, A and B, resemble each other in a number of attributes, p, q, r, s. A has an additional attribute t. That is, the attributes p, q, r, s, t are found coexisting in A. If t is causally connected with the other attributes, t will be present also in B. The inference will then be certain and amount to an induction, being true not only of B but also of any other individual belonging to the class of A and possessing the attributes p, q, r, s, t. If there is no causal connection between t and p, q, r, s, the inference will be probable, the degree of probability depending on the nature and number of the qualities in which B resembles A. If p, q, r, s are fundamental qualities of A and B, then t, which is found coexisting with them in A, will most probably be found coexisting with them in B. If p, q, r, s, are only superficial qualities, then the probability of t being found coexisting with p, q, r, s in B because it occurs with them in A will be much less. We have examples of such inferences in Zoology and Botany. "A fish resembles an amphibian (say a frog) in the possession of a backbone, a spinal cord and brain, and a heart; the amphibian has organs of sense: the fish has therefore the organs of sense. The former has limbs; the latter has therefore limbs. The inferences here are analogical and should be verified by observation. The inference "the fish has organs of sense" similar to those of the

amphibian is found to be true, while the inference it has limbs is not quite true, because instead of limbs like those of the frog it has fins which correspond to them and perform the function of locomotion, but are not like them in form and structure. The fossilised remains of an extinct animal resemble a living animal in, say, the possession of a backbone, a skull and limbs; it may be inferred by analogy that the extinct animal resembled the living animal in the possession of such other organs as a heart, a nervous system and organs of sense, muscles and tissues as are possessed by the latter. Two plants resemble each other in the essential parts of their flowers as well as in their leaves, branches and general appearance; therefore they will resemble also in their fruit. This is an inference by analogy The fruit of one, as in the case of mango trees, may resemble that of another in form and size; but the quality of one may be quite different from that of the other. A bird and a bat have flying organs and other superficial resemblances. What is true of one is not true of the other, because they differ in the more important and fundamental organs in virtue of which they belong to different classes of the backboned animals, the bat being a mammal.

§ 4. (iii) Inductions of Succession or Causation.

Whenever a causal connection can be established between two phenomena, the relation between them gives rise to a valid induction. According to Mill the cause of a phenomenon is the sum of positive and negative conditions which being realised, the phenomenon invariably and unconditionally follows. Take the phenomenon of lighting a match by rubbing it on the surface of the box. The cause of the phenomenon of a lighted match is the friction of a chemically prepared end of a little stick on the chemically prepared surface of the match-box. Here the phenomenon is a luminous flame, and its cause is a positive condition, viz. the presence of certain circumstances and a negative condition, viz.

the absence of certain other circumstances. The positive condition consists of (1) a small stick duly prepared with necessary chemicals, (2) a chemically prepared surface of the match-box, (3) the skilful rubbing of the prepared end of the stick on the prepared surface, (4) the usual atmospheric condition. The negative condition consists of (1) the absence of damp in the atmosphere, in the stick-end and on the prepared surface, (2) the chemical preparation of the end of the stick and of the surface of the box not having in the meantime disappeared or been badly affected, (3) the rubbing not being on a part of the surface which has, by previous use or otherwise, lost its chemical properties. In other words, the cause of the flame is the assemblage of the positive conditions, all counteracting circumstances being absent.

These positive and negative conditions being fulfilled, the effect, namely, a luminous flame at one end of the stick, follows. The effect will not follow if any of the conditions which must be absent are present or if any of the positive conditions are absent. In the example given above, the rubbing would be popularly regarded as the cause of the phenomenon of lighting the match, though the chemical preparations of the stick and the surface are essential conditions of it. If the match is not lighted by the rubbing, the cause would popularly be considered to be the dampness of the air—the absence of which is one of the negative conditions. There is thus great difference between Mill's definition of the term Cause and its popular use. It has been pointed out that it is desirable to use the term Cause neither in Mill's sense nor in the popular sense, but in a sense intermediate between the two, that is, to mean the sum of the minimum conditions which being given, the effect invariably follows. In the example of the lighting of a match, the minimum conditions would be the chemical preparations of the stick and the surface plus the rubbing. These therefore should be usually regarded as the

cause of the phenomenon, the conditions being severally regarded not as a cause of it but as essential or indispensable to it.

§ 5. Mill regards the Law of Causation as the most certain law of Nature. Though he proves it by the method of Simple Enumeration, he thinks that there are no exceptions to it. All our experience, he says, confirms it. If there were any exceptions, our experience would discover it. Scientific men in dealing with a phenomenon proceed on the assumptions (1) that it has another phenomenon or group of phenomena for its antecedents, and (2) that the same antecedents will always produce the same effect. When they are unable to trace a phenomenon to its antecedent, they try to find out the phenomena with which it co-exists and also the phenomena which it resembles. The phenomena coexisting with the one under investigation will give rise to an induction or law of coexistence. In the example given above, the phenomena coexisting with the given phenomena of a lighted match are (1) the heat of the flame, (2) the smoke which escapes from the flame, (3) the rise in temperature of the air in contact with the flame, (4) the smell of the gases produced, etc. All these phenomena coexist in the effect and form parts of it. In a scientific inquiry, therefore, the object is to find the phenomena connected with a given phenomenon under investigation. The connected phenomena may be those coexisting with the given phenomenon and thus suggesting the class of phenomena to which it belongs, or those preceding it. In the former case a law of Coexistence and along with it a law of Resemblance will be discovered, and in the latter case a law of Succession which may be shown to be a law of Cause and Effect. It may be further shown that the phenomena coexisting are joint effects of a common Cause; and a law of Coexistence may be thus resolved into a law of Succession or Causation. The light and heat of the flame are

coexisting phenomena in the above example and are joint effects of the common cause, namely, the rubbing of the stick-end on the surface of the box.

The object of scientific investigation is to discover and prove the laws of Coexistence, Resemblance and Succession and to resolve them into laws of Causation among phenomena.

§ 6. Inductions have been distinguished into (1) Perfect and (2) Imperfect. A perfect induction is the inference of a general proposition after the examination of every individual belonging to a class. An imperfect induction is the inference of a general proposition after the examination of some individuals of a class. In the former, what is already known to be true of every individual belonging to a class is inferred of the whole class. In the latter what is known to be true of some individuals of a class is inferred of the whole class. Every known planet is observed to go round the sun: the inference, from the particular observations of these known planets, of the general proposition "all the known planets go round the sun," is a perfect induction, while the inference from the same observations of the general proposition "all planets go round the sun" is an imperfect induction. The former applies only to the planets that are known to us, while the latter applies also to those that may be discovered afterwards. "All metals conduct heat and electricity" is an imperfect induction as it applies to the elements known now to be metals and also to those that may hereafter be discovered to be metals, while the proposition "all the metals conduct heat and electricity" would be a perfect induction if it was inferred after the examination of all individual known metals and applied to them only; but it would be an imperfect induction, if it was inferred from the examination of some individual metals and applied to all the known metals. According to Mill an induction proper is an inference from the known to the unknown,

from the observed to the unobserved on the ground of the Uniformity of Nature. In the case of a perfect induction there is no such transition; there is, according to him, no inference at all. The so-called general proposition or induction is merely a summary of the individual observations. A perfect induction is not, according to him, a generalisation but merely a summary statement of the particulars already known individually. It is true that, in a perfect induction, there is no inference from some to all individuals of a class as in the case of an imperfect induction, but in both there is reliance on the Uniformity of Nature. That all the known planets will continue to move round the sun, that all the known metals will retain all their properties and continue to conduct heat and electricity, are inferred on the ground of the Uniformity of Nature and the perfect inductions may therefore claim to be recognised as inductions. In an imperfect induction, the principle of the Uniformity of Nature forms the ground of the transition from some to all as well as from the present to the future. Both perfect and imperfect inductions therefore depend for their validity on the Uniformity of Nature. The inference that the sun will in future daily rise in the east and set in the west is an induction drawn from the instances of its rising in the east and setting in the west. That a particular tree will in future years bear fruit of the same quality as it has done in the past is an induction founded on the Uniformity of Nature. It will bear fruit of the same quality if the conditions of the soil and the atmosphere continue the same as before, but if either of these conditions changes, the fruit of the tree may be of a much inferior quality. The induction is drawn from the instances of its bearing fruit of the same quality in past years. We thus have inductions in regard to an individual thing as well as a class.

CHAPTER VI.

The Methods of Induction.

§ 1. The different methods by which inductions are established are

(1) The Method of Simple Enumeration,

(2) The Methods of Elimination by which the unnecessary circumstances can be excluded and the necessary or relevant circumstance or circumstances discovered as cause of a given effect or as effect of a given cause. Mill's methods of experimental inquiry are methods of elimination.

Given a phenomenon, it is required to find other phenomena connected with it. The phenomena connected with it may be (1) those coexisting with it, (2) the antecedents or the consequents of it. In the second case, the phenemenon is given as an effect and it is required to find its cause; or it is given as a cause and it is required to find its effect. In the first case, it is required to find the phenomena coexisting with the given phenomenon, no distinction being made between antecedents and consequents.*

§ 2. The Method of Simple Enumeration.

The Method of Simple Enumeration is the most ancient as well as the most popular method of induction. The earliest knowledge of mankind in regard to natural phenomena and human life and society is founded on this method. Children acquire their knowledge of things from a very early period of their life by this method. They instinctively believe that what has happened once will happen again under the same circumstances. The burnt child dreads the fire.

The Method of Simple Enumeration consists in inferring a general proposition from instances of a phenomenon, observed in Nature or in human affairs, without any systematic attempt at the analysis of the phenomena and the discovery of the true cause or the true effect by the elimination of the circumstances which are irrelevant or unnecessary. It does not seek and consider the negative instances, that is, instances in which the phenomenon is absent. It simply states the instances in which the phenomenon occurs and generalises on the basis of those instances. The validity of the induction thus drawn depends on the number of the instances observed and the possibility or probability of there being contrary instances. If the induction, as in the case of the proposition "all animals are mortal," is confirmed by all our experience, it is accepted as true. If, on the contrary, it is contradicted by some contrary instances as in the case of the proposition "all plants have flowers," it is modified and limited in extent.

§ 3. Symbolical Statement of the Method of Simple Enumeration.

(1) Let the given phenomenon be represented by A and the phenomena or circumstances coexisting with it by B, C, D, E &c. Observe the phenomenon A in a number of instances in which it happens, and let the instances be represented as follows without any distinction of antecedents and consequents:—

$$\begin{array}{ccccc} A & B & C & D & E \\ A & F & G & H & E \\ A & K & L & M & E \end{array}$$

These instances prove that A and E are connected: both may be joint effects of a common cause or one may be cause and the other effect. Other instances of A may be observed and the connection of A with E confirmed by them.

(2) Observe the phenomenon A in a number of in-

stances in which the antecedents are distinguished from the consequents, the former being represented by capital letters and the latter by small ones.

Let the instances be as follows;—

$A\ B\ C$	$p\ q\ r$
$A\ B\ D$	$p\ q\ s$
$A\ D\ E$	$p\ s\ t$
etc.	etc.

Here A is the antecedent given and p the consequent found connected with it in the above instances: p is therefore an effect of A. In the reverse case, in which the phenomenon given is a consequent, represented by p, the above instances will indicate that A is a cause of the effect p.

The instances in both cases may be multiplied without any attempt at elimination or exclusion of unnecessary or irrelevant consequents in one case, and of unnecessary or irrelevant antecedents in the other case. Other instances may be observed and may be represented as follows :—

$A\ B\ K$	$p\ q\ n$
$A\ C\ L$	$p\ r\ n$
$A\ E\ H$	$p\ t\ v$
etc.	etc. ;

A standing as before for the given antecedent phenomenon in one case and p for the given consequent phenomenon in the other case.

§ 4. Concrete Examples.

(1) The given phenomenon is the presence of flowers in plants; it is required to find the other phenomena with which it is connected. Taking some instances of higher plants, it is found that the phenomenon of flowers is connected with the phenomena of the presence of leaves, branches, roots in the same plants and the induction drawn from them is that all these phenomena co-

exist in all higher plants. Do they coexist in all plants? The instances of lower plants such as ferns and mosses, which are flowerless, show that those phenomena do not coexist in all plants. Are they connected causally? Are flowers cause or effect of roots, branches and leaves? Are they all joint effects of a common cause? These questions come under the investigation of a Botanist. The popular answer would be that all those parts of a plant are connected and produced alike by the plant which is the result of the germination of a seed. At first only a little root and two leaves are produced from the seed under proper conditions of soil, air, light and heat, and this little plant grows and gradually produces all the parts including the trunk.

(2) The given phenomenon is the backbone of an animal: it is required to find out the other phenomena with which it is connected in an animal.

Taking some instances of animals having a backbone, it is found that it is connected with a head, limbs, a heart, a nervous system and organs of sense, etc., and the induction is drawn from these instances that these parts coexist in all animals possessing a backbone. On further observation it is found that all fishes, all amphibia (frogs, etc.), all reptilia (snakes etc.), all birds and all mammalia including man have these parts coexisting in every individual. Are they causally connected? Is the backbone the cause or effect of the others? Are they joint effects of a common cause? These questions come under the investigation of the comparative Anatomist and Embryologist. The popular answer would be on the same lines as in the preceding example.

(3) Given the yellow colour of a piece of gold as the phenomenon under investigation. The connected phenomena would be the chemical and physical (other than the colour) qualities of gold possessed by each sample. They all coexist with the yellow colour in every molecule of gold. They are, as we say, inherent in it

and cohere in the substance gold. Are they causally connected? Can we say some are cause and some others are effect? Are they all joints effects of a common cause? These questions belong to the sciences of Physics and Chemistry.

(4) Take the phenomenon of a lighted spirit lamp given as an antecedent and observe the instances of its heating and expanding water, oil, mercury, etc., in a flask. Here the antecedent phenomena are the lighted spirit lamp, a liquid (water, oil, mercury), and a flask. The consequent phenomena are the liquid expanded in the glass flask over the lighted spirit lamp. From these instances, it may be inferred that the expansion of the liquid is the effect of the given antecedent, the lighted spirit lamp. Taking the expansion of the liquid as the given phenomenon, it may be inferred that the lighted spirit lamp is the cause of it. The induction is that heat expands liquids. Another phenomenon common to all the consequents, which could be easily observed by touch or by a thermometer, is the rise of temperature of the liquid. This is also an effect of the given cause, the lighted spirit lamp. The induction is that heat raises the temperature of liquids. What is the relation of the two effects, expansion and rise in temperature of the liquids, to each other? Are they co-existent or one antecedent and the other consequent? They take place simultaneously and appear to be joint effects of the same cause, heat.

§ 5. According to Mill the Laws of Thought, the Axioms of Geometry, the Principles of the Uniformity of Nature and of Causation, etc., are generalisations from experience by this method. They are regarded and accepted as universally true because there are no exceptions to them, because they are confirmed by all our experience. (1) The axiom that things equal to the same thing are equal to one another, is a generalisation from the innumerable instances in which we find that two things equal to

the same thing are equal to each other. (2) The Law of Thought, that a thing cannot have two contradictory attributes—that *A* cannot at the same time be *B* and not-*B*—is a generalisation from our experience that *B* and not-*B* are never found coexisting in one and the same subject, that the presence of *B* in a thing means the disappearance of not-*B*, and the presence of not-*B* the disappearance of *B*, that the two ideas *B* and not-*B* cannot in any case exist together. (3) The proposition that every phenomenon is preceded by another phenomenon, is a generalisation by this method. In all instances of phenomena in the external world, we find that a phenomenon has always another phenomenon for its antecedent. The induction is confirmed by all our experience of physical phenomena. It forms the basis of Mill's Law of Causation which he applies not only to all physical but also to mental phenomena.

CHAPTER VII.

(2) THE METHODS OF ELIMINATION.

§ 1. Mill's Methods of Experimental Inquiry are methods by which the cause of a given effect or the effect of a given cause is determined by the elimination of the irrelevant or unnecessary circumstances.

Let p be a given effect; it is required to find its cause.

Observe or produce by experiment an instance in which p occurs. Analyse the antecedent circumstances of the instance and represent them by A, B, C. In this case the cause of p may be the aggregate of the three circumstances $A\ B\ C$ or a combination of any two of them, $A\ B$, or $A\ C$, or $B\ C$, or any single circumstance A, or B, or C. The effect p may be of such a nature that it requires the interaction of all the three circumstances to produce it. On this supposition the cause of p will be the sum of the circumstances $A\ B\ C$. Or the effect p may require the interaction of only two circumstances and these may be either $A\ B$, or $A\ C$, or $B\ C$. Or the effect p may be produced by one circumstance only, and this may be A, or B, or C.

§ 2. In Nature we have instances of phenomena which are produced in the several ways noted above. For example, the sensation of light is produced by (1) a luminous object, (2) the organ of sight, and (3) the faculty of seeing. These three are necessary to produce any sensation of light. They must therefore be taken together and regarded as the cause or indispensable conditions of the effect—a sensation of light.

The growth of a plant by the germination of a seed is brought about by the interaction of agents such as the

soil, the moisture, the heat and light of the sun, the air of the atmosphere, the vitality of the seed itself—all of which should be taken together and regarded as cause or indispensable conditions of the phenomenon. In chemical actions, the elements producing a compound must jointly be regarded as the cause of the compound produced. The effects in these cases are quite unlike the effects of the individual circumstances or agents which jointly by their interaction produce them. There are, again, instances of phenomena in which the effect is the sum of the effects of the several antecedents which jointly produce it. In these cases each of the several antecedents produces the same kind of effect. The simplest example of such cases is the action of two forces on a body in the same direction, producing an effect which is the sum of the effects of the two forces. Two forces acting on a body in opposite directions will produce an effect which is the difference of the effects of the two forces. Two forces acting on a body at an angle will move the body in a direction which can be determined by the parallelogram of forces, the sides representing the two forces and the diagonal representing the resultant motion as the effect. A liquid may be heated by three different lamps of spirit, kerosine and mustard oil; the effect will be the sum of the effects produced by them, as each antecedent produces the same sort of effect, namely, the expansion and rise in temperature of the liquid. A room may be lighted by ten candles, the effect produced by each candle being a certain degree of luminosity; the total effect is the sum of the effects produced by all. In these cases, some of the antecedents may be removed without changing the nature of the effect.

In determining the causes of phenomena, it must, therefore, be remembered that the cause of a phenomenon may consist either of antecedents which jointly produce an effect which is quite different from the effect of each of them, or of antecedents which produce an effect which

is of the same nature as the effect of each of them. Mill calls the former Chemical or Heteropathic Intermixture of effects and the latter Mechanical or Homogeneous Intermixture of Effects. In the Mechanical, the effect is the sum of the effects of the several antecedents; and the cause is equivalent to the effect. To this fact is due the idea that the cause and its effect are identical or equal. This is true in the causation of the phenomena of Physics where all the causes produce some sort of motion. This is hardly true in Chemistry, and much less in Biology and not at all in Psychology. In Chemistry the weight of the constituent substances taken together is found, by experiment, to be equal to the weight of the compound produced; and this is the only property in which they agree: in other respects the properties of the compound are quite different from those of the constituent substances. For example, the properties of water are quite different from those of hydrogen or oxygen. In Biology, the form and the qualities of the plant produced by the germination of the seed are quite different from those of the antecedent agents. So are also the forms and qualities of the leaf, the flower, the fruit, etc., which the plant gradually produces, from those of the ingredients of the soil and atmosphere, under the influence of the sun. In Psychology, the sensation of light is quite unlike the antecedents. So are also other sensations unlike their antecedents. The mental phenomena are quite different from their physical antecedents. As biological phenomena are to a great extent determined by the vitality and nature of the seed, so are also the phenomena of the mind determined mainly by the nature and faculties of the mind itself. Some philosophers maintain that they are entirely due to the mind itself and that the relationship between the mental phenomena and the corresponding physical antecedents is merely one of concomitance, while others hold that there is interaction between them—that in sensation, for

example, a psychical phenomenon is produced in the mind by some physical antecedents and that in voluntary action, on the other hand, a physical phenomenon is produced by some mental antecedents. In any case the mind itself must be regarded as an essential condition of every mental phenomenon.

§ 3. Given an effect p, it may be produced by several distinct causes. Heat, for example, may be produced by mechanical action such as rubbing or by chemical action such as the union of hydrogen with chlorine. Motion may be produced by steam or by electricity. Death may be caused by various diseases. It may be brought about by the failure of the heart, by the decay of the lungs or by the stoppage of the function of some vital part of the brain. Sensations may be produced by external objects or by subjective causes, that is, by changes in the brain or in the organs of sense of the organism itself. Germination of a seed may be brought about by artificial means as well as by natural causes. There are thus many instances of phenomena where an effect of the same kind is produced by causes which are distinct and separable from one another. In one instance the effect is produced by one cause and in another instance the same effect is produced by another cause quite distinct from the first. There is thus in Nature the fact of a plurality of distinct causes of an effect of the same kind. This fact must be recognised and allowed for in applying the Methods of Elimination for the discovery and proof of the cause of a given effect.

It has been pointed out that these cases of a plurality of causes of a given effect arise from our overlooking the collateral circumstances of the given effect. If all the consequent circumstances are taken into consideration, it would be found that the same consequents are always produced by the same antecedents, that the given effect p is only one of the many circumstances which together form the complex effect of the many

antecedent circumstances which together constitute the complex cause. Death by heart failure has, for example, collateral circumstances which are different from those of death by the decay of lungs or by some other cause. The phenomenon of death is common to the several instances of dying; but the other circumstances connected with it are different in each case. The phenomenon of the production of heat is likewise common to the different instances in which heat is produced by mechanical or by chemical action, but the other circumstances connected with it are different in each case. So the fact of a plurality of causes of the same effect is due to our overlooking all the circumstances of a complex consequent, except that one which is given as an effect and which is present not only in the instance under consideration but in many other instances.

§ 4. The Canons of Mill's Methods of Experimental Inquiry.

Mill's experimental methods of inquiry proceed on the assumption of the truth of the two fundamental principles of Causation and Uniformity of Nature. They are conducted according to certain laws or rules which he calls the Canons of the Methods, In Appendix D of my Text-book of Deductive Logic, I have shown that the canons follow from the two laws of Causation and Uniformity of Nature.

The law of Causation implies the two propositions—(1) every phenomenon has a cause and (2) the cause of a phenomenon is the invariable or, as Mill says, the unconditionally invariable antecedent of the phenomenon. The law of Uniformity of Nature means that (3) the same cause or antecedent will, under the same circumstances, produce the same effect or consequent. From the second proposition of the first law follows (4) "whatever antecedent can be left out, without prejudice to the effect, or whatever antecedent can be present, without the effect being present, can be no part of the cause"; (5)

"when an antecedent can not be left out, without the consequent disappearing, such an antecedent must be the cause or part of the cause"; (6) "an antecedent and a consequent rising and falling together in numerical concomitance are to be held as cause and effect"; (7) "if two or more instances of a phenomenon under investigation have only one circumstance in common, that circumstance is the cause (or effect) of the phenomenon"; and (8) "if an instance where a phenomenon occurs and an instance where it does not occur have every circumstance in common except one, that one occurring only in the first, the circumstance present in the first and absent in the second, is the cause or a part of the cause of the given phenomenon."

The proposition marked (7) is the canon of the Method of Agreement. The proposition marked (8) is the canon of the Method of Difference. The proposition marked (6) is the canon of the Method of Concomitant Variations. The joint Method of Agreement and Difference is really a method of agreement both in presence and absence. The canon consists of the two propositions marked (7) and (5). The Method of Residues is an experimental method which is aided by the Method of Deduction. The propositions marked (4) and (5) are the rules respectively for the elimination of irrelevant and unnecessary circumstances and for the detection and proof of the cause of a given effect.

CHAPTER VIII.

THE METHOD OF AGREEMENT AND THE JOINT METHOD.

I. Method of Agreement.

§ 1. The canon of the Method is:—If two or more instances of a phenomenon under investigation have only one circumstance in common, that circumstance is the cause (or effect) of the phenomenon.

Mill proceeds on the assumption that the phenomenon under investigation, if it is an effect, has only one distinct cause and that it is not the result of an intermixture of effects of several antecedents. He further assumes that the antecedents and the consequents of an instance of the phenomenon under investigation may be analysed into circumstances which are simple and separable from one another.

Let the phenomenon under investigation be an effect represented by p and two instances of it be represented as follows:—

(1) $A\ B\ C$ .. .. $p\ q\ r$
(2) $A\ D\ E$ .. .. $p\ s\ t$

where $A\ B\ C$ are the simple antecedents and $p\ q\ r$ the simple consequents of one instance; and $A\ D\ E$ the simple antecedents and $p\ s\ t$ the simple consequents of the other. The given phenomenon p as an effect is present among the consequents of both the instances, $q\ r$ being the collateral circumstances in one and $s\ t$ in the other.

From (1), $D\ E$ cannot be any part of the cause of p, according to proposition (4) marked in Chapter VII, Section 4, that is, the rule, "Whatever antecedent can be

left out, without prejudice to the effect, can be no part of the cause."

From (2), B C cannot be any part of the cause of p, according to the same rule.

From (1) and (2), it may therefore be inferred that the one simple antecedent circumstance, A, which is common to both the instances, is the cause of p, according to the canon of this Method.

This inference is true (1) if p has only one distinct cause and (2) if it is not the result of an intermixture of the effects of several antecedents.

§ 2. Plurality of Causes and Intermixture of Effects.

(1) If p has a plurality of distinct causes, there may be two other instances of it represented as follows:—

(1) $F\ G\ H$ $\quad p\ l\ m$
(2) $F\ I\ K$ $\quad p\ n\ o$

From those two instances it may be inferred that F is a cause of p.

There may be other instances where p is produced by other distinct causes.

(2) If p is the result of an intermixture of the effects of several antecedents, then a combination of any two antecedents A B, or A C or B C, or the aggregate of all the three antecedents A B C may be the cause of p in the first instance of the first example. In the second instance of the same example, A D, or A E, or D E, or A D E, any one of these combinations of antecedents, may be the cause of p. In such a case the Method of Agreement is not applicable. The cause of the phenomenon cannot be found by this method. There will then be no one circumstance common to the antecedents of all the instances of the phenomenon. Suppose p is the result of the intermixture of the effects of A B, then there may be two instances of it as follows:—

(1) $\overline{AB}\ C\ D$ $\quad p\ q\ r$
(2) $\overline{AB}\ E\ H$ $\quad p\ s\ t$

From (1) and (2), it may be inferred that $\overline{AB}$ is the cause of p. If p has a plurality of distinct combinations of antecedents as causes, then there may be instances of it as follows:—

$$(1)\ \overline{FG}\,H\ K \qquad p\ l\ m$$
$$(2)\ \overline{FG}\,I\ L \qquad p\ n\ o$$

From (1) and (2), it may be inferred that $\overline{FG}$ is the cause of p.

Thus p may have a plurality of causes also in the case of its being the result of an intermixture of effects.

Mill points out that when the phenomenon under investigation is the result of a homogeneous or mechanical intermixture of effects of several antecedents, the investigation of its cause or causes should be conducted according to the Deductive Method.

It may be pointed out that when the phenomenon under investigation is the result of an heterogeneous or chemical intermixture of effects of several antecedents, its cause may be investigated by the Method of Agreement. The one common circumstance among the antecedents of the phenomenon will then be not simple but compound or complex, consisting of several antecedents, as for example, in the case of the sensation of light, or the germination of a seed, or the production of water from the union of hydrogen and oxygen.

When p has several distinct causes, for example, A, F, etc., it may be found on careful scrutiny of A, F etc., that each of them is complex, consisting of two or more simpler circumstances; and that among these there is some circumstance which is common to them all. This will be then a case of intermixture of effects, as p will be produced by the circumstance common to A, F, etc., plus the circumstance or circumstances in which they differ. It may be that the common circumstance is a merely nominal antecedent and cannot be regarded as

the cause of p, though such an inference would be justified by the Method of Agreement. Whether such an antecedent is the cause or not, may be verified by the Method of Difference.

§ 3. Examples:

1. Cold applied to water in an iron vessel freezes it. Cold applied to cocoanut oil in a glass bottle freezes it. Therefore cold is the cause of freezing. The phenomenon under investigation is that of freezing. Cold is the one common circumstance present in the antecedents of the two instances of the phenomenon and is therefore its cause.

2. Heat applied to the frozen water in the iron vessel melts the water. Heat applied to the frozen cocoanut oil in a glass bottle melts the oil. Therefore heat is the cause of melting.

3. Attention paid to a perception leads to its retention in memory. Attention paid to an idea leads to its retention in memory. The phenomenon under investigation is that of retention in memory of a mental state. Attention is the common circumstance in the antecedents of the two instances of the phenomenon. Therefore attention is the cause of the phenomenon of retention.

4. Prove the following propositions by the Method of Agreement:—

(i) Heat is the cause of the expansion of material bodies.

(ii) A luminous object is a condition of the sensation of light.

(iii) An organ of sense is a condition of a sensation.

(iv) Contiguity is a cause of the association of mental phenomena.

(v) Resemblance is a cause of the assimilation of mental phenomena.

II. The Joint Method of Agreement and Difference or the Method of Agreement both in Presence and Absence.

§ 4. The Canon of this Method is :—"If two or more instances of a phenomenon under investigation have only one circumstance in common, while two or more instances in which the phenomenon does not occur have nothing in common except the absence of that circumstance, the circumstance in which the two sets of instances differ is the cause (or effect) of the phenomenon, provided both the sets of instances belong to the same department of inquiry."

Let p, an effect, be the phenomenon under investigation and the instances in which it occurs be represented as follows :—

I. Positive instances—

$A\ B\ C$	$p\ q\ r$
$A\ D\ E$	$p\ s\ t$
$A\ F\ G$	$p\ u\ v$

Let the instances in which p does not occur be represented as follows :—

II. Negative instances—

$C\ H\ F$	$r\ w\ x$
$B\ D\ K$	$q\ y\ z$
$E\ G\ M$	$t\ h\ n$

From the first set of instances it may be inferred by the Method of Agreement that "A is the cause of p," under conditions which have been discussed under that Method. This inference is confirmed by the second set of instances in which both A and p are absent, and which therefore show according to rule 4 given in Chapter VII, Section 4, that the antecedents B, C, D, E, F, G which are present in one or other of the three positive instances as well as of the three negative instances, without the effect p being present in the latter, cannot be the cause of p. The inference drawn from the first set is confirmed by the second set, by excluding all the antecedents except A in the positive instances from the category of the

cause of p. There is thus agreement of A with p in both presence as in the positive instances and in absence as in the negative instances. The negative instances should be selected from the department of inquiry to which p belongs; and if they could be made exhaustive in that department, the inference would be of great value. In the symbolical example given above the common letters show that the two sets of instances are taken from the same department. But it would still be true only under the conditions discussed under the Method of Agreement.

§ 5. Examples:

Prove the following propositions by the Joint Method:—

(1) Cold is the cause of the freezing of water.
(2) Heat is the cause of the melting of ice.
(3) A luminous object is an indispensable condition of a sensation of light.
(4) The faculty of hearing is an indispensable condition of a sensation of sound.
(5) Attention is the cause of the retention of a mental state.
(6) Contiguity is a cause of the association of mental phenomena.
(7) Resemblance is a cause of the assimilation of mental phenomena.

CHAPTER IX.

III. THE METHOD OF DIFFERENCE.

§ 1. The Canon of the Method is :—"If an instance where a phenomenon occurs and an instance where it does not occur have every circumstance in common except one, that one occurring only in the first, the circumstance present in the first and absent in the second is the cause, or a part of the cause, of the given phenomenon."

Let the given phenomenon be p and the two instances be represented as follows :—

$$\begin{array}{llll} (1) & A\ B\ C & \ldots\ \ldots & p\ q\ r. \\ (2) & \ \ B\ C & \ldots\ \ldots & \ \ q\ r. \end{array}$$

Here the two instances, in the first of which p occurs and in the second it does not occur, have every circumstance in common, except A: the circumstance A, occurring only in the first and absent in the second, is, therefore, the cause or a part of the cause of p.

Taking the example of lighting a match, p would be the flame, $q\ r$ its collateral effects, A the rubbing and $B\ C$ the other antecedent circumstances, 'the stick and the surface. The two instances would be (1) the antecedents and consequents after rubbing when the effect, the flame, is produced, and (2) the antecedents and consequents before rubbing. They agree in every circumstance except one, namely, the circumstance of rubbing; this circumstance is therefore the cause of the flame. This is a popular statement of the circumstances; and the rubbing is regarded as the cause of the effect. In a scienti-

fic statement it would be necessary to follow the sequence of the phenomena more closely. The rubbing produces a little heat which brings about a union of the chemical substances of the stick and the surface, producing some gaseous products and more heat. This heat raises the temperature of the gases, causing the kindling of the stick and the flame. Between the rubbing and the flame there are several phenomena connected as cause and effect. The flame is a remote effect of the rubbing. The immediate effect of the rubbing is the production of the little heat which again acts as an antecedent and produces along with other antecedents certain effects which bring about the kindling of the stick. Scientifically, therefore, the inference which can be drawn from the two instances is that the rubbing is the cause of the little heat which is necessary to bring about the union of the chemical substances and is only a remote condition of the flame. This concrete example shows that the two instances should be, strictly speaking, in immediate succession—that there should be no lapse of time and no other links between them, and that they should be identical in all respects except in the antecedent circumstance which produces the effect; and that this effect is the immediate change brought about by the antecedent and not the remote consequent which may be produced by the effect acting as an antecedent along with other antecedents as a cause. The flame is such a remote consequent and is produced by the conjoint action of several antecedents. Each of these antecedents may be proved by the Method of Difference to be a part of its cause.

In the case of a sensation of light, the physical conditions are (1) a luminous object and (2) the eyes with the nerves, and brain, and the mental conditions are (3) the faculty of vision, and (4) attention; and it can be proved by the Method of Difference, that each of these conditions is a part of the cause of the phenomenon of the

sensation of light. When all these conditions are fulfilled a sensation of light is produced; and when any one of these conditions is wanting the sensation is not produced.

In the case of the germination of a seed, it may be proved by the Method of Difference that the conditions of the soil, the moisture, the heat and light of the sun, and the atmospheric air are parts of the cause of the phenomenon. Each of these may be alternately supplied to, and removed from, the seed and the effect observed. The difficulty lies in making the experiments, securing all the conditions required.

In the examples given above, the effect p is the result either of a single antecedent as in the case of the heat produced by the rubbing or of an heterogeneous intermixture of the effects of several antecedents. In the former case the single antecedent is the cause, and in the latter each of the several antecedents is a part of the cause of the phenomenon. In the symbolical statement of the latter, A would be a complex group of antecedents, and p an effect which is produced by the conjoint action of the several antecedents. The positive instance in which the effect is present would be represented as follows:—

$$\overline{ABC}\ \ D\ E \quad \ldots \quad p\ q\ r$$

where three different antecedents conjointly produce the effect p, $D\ E$ being the other antecedents and $q\ r$ being the other consequents. A negative instance in which p is absent would be as follows:—

$$\overline{BC}\ D\ E \quad \ldots \quad \ldots \quad q\ r$$

proving that A is a part of the cause of p, or as follows:—

$$\overline{AC}\ \ D\ E \quad \ldots \quad \ldots \quad q\ r$$

proving that B is a part of the cause of p, or as follows:—

$$\overline{AB}\ \ D\ E\ ..\qquad\qquad q\ r$$

proving that C is a part of the cause of p.

These three negative instances with the one positive instance would respectively prove that each of the three antecedents A, B, C is a part of the cause of p, while a single negative instance $D\ E\ \ldots\ q\ r$, with the positive instance $A\ B\ C\ D\ E \ldots . p\ q\ r$ would prove that the three antecedents A, B, C are jointly the cause of p.

§ 2. If p is the result of a homogeneous intermixture of the effects of several antecedents, then its cause cannot be found by the Method of Difference. Let p be the sum of the effects of A and B, $q\ r$ the collateral consequents and $C\ D$ the collateral antecedents, then the instance in which p is present will be represented as follows :—

$$\overline{AB}\ \ C\ D\ ..\qquad\qquad ..\ \ p\ q\ r$$

If A is absent in the second instance and B is present in it, p will be still partly present in it as p is the result of the sum of the effects of both A and B. Thus the negative instance in which p is entirely absent will not be available ; and the Method of Difference will be inapplicable in such cases. Mill points out that such cases should be investigated by the Deductive Method.

If $A\ B$ be taken together as a complex or compound antecedent consisting of two conditions or circumstances, then $A\ B$ may be inferred to be a cause of p from these two instances :—

$$\begin{array}{llll} \overline{AB} & CD\ .. & .. & p\ q\ r \\ & CD\ .. & .. & \ \ q\ r. \end{array}$$

But in order to find the effect of A, it would be necessary to deduct from p the effect of B ; and this would be done according to the rules of the Deductive Method : that is, the effect of each of the two antecedents being known by previous inductions, the effect of their interaction would be determined by the Deductive Method.

§ 3. Examples:

(1) Heat applied to water in a vessel raises its temperature and is therefore the cause of the rise of its temperature. The two instances are (1) heat applied to water in a vessel raising its temperature and (2) water in a vessel without any heat being applied to it. The phenomenon under investigation is the rise of temperature of the water. It is present in the first and absent in the second. The one circumstance, application of heat, present in the first and absent in the second is, therefore, the cause or a part of the cause of the rise of temperature. The two instances agree in everything except in the one circumstance, application of heat.

(2) An external object is an indispensable condition of a sensation. This proposition can be proved by the Method of Difference in the case of the several senses, (1) by having an appropriate external object affecting the special organ of sense of a person and his attending to it in one instance (positive) and (2) by removing the object in the other instance (negative) so that his organ of sense may not be affected by it. From two such instances it may be inferred that the object is a cause or a part of the cause of the sensation.

(3) By similar observations or experiments it may be inferred that attention on the part of a person is an essential condition of the consciousness of a sensation. His organ of sense, say, that of hearing, may be affected by a sound; but he will not be conscious of it if he is deeply attending to something else, that is, the consciousness of a sensation of sound depends on his attending to it. A student intently listening, say, to a lecture in the class, does not often hear the bell. A person deeply absorbed in study does not hear even when he is called loudly.

(4) Prove the following propositions by the Method of Difference :—

(i) Cold is the cause of freezing.

(ii) Heat is the cause of the melting of ice.
(iii) A special organ of sense is an indispensable condition of sensation.
(iv) Attention and repetition are the causes of the retention of a mental state.
(v) The development of a faculty depends on its exercise.
(vi) Life is the cause of the organisation of a body.
(vii) Mind is the cause of feelings.

CHAPTER X

IV. The Method of Concomitant Variations.

§ 1. The Canon of the Method is :—" An antecedent and a consequent rising and falling together in *numerical concomitance* are to be held as cause and effect."

A more accurate statement would be as follows :—

Whenever an antecedent and a consequent of a phenomenon vary together in a certain manner either directly or inversely, that is, both increasing together or one increasing and the other decreasing, the antecedent is either the cause, or a part of the cause, of the consequent.

A more comprehensive statement would be as follows :—

Two circumstances of a phenomenon varying together in a certain manner, that is, either increasing together or one increasing and the other decreasing, are causally connected, that is, connected as cause and effect or as joint effects of a common cause.

Let A be an antecedent and p a consequent, the two rising and falling together as represented in the following instances :—

I.

(1)	$A\ B\ C$ ..	.. $p\ q\ r$
(2)	$2\ A\ B'\ C'$ ..	.. $2\ p\ q'\ r'$
(3)	$3\ A\ B''\ C''$	.. $3\ p\ q''\ r''$
	etc.	etc.

In these three instances, p increases as A increases : A is therefore the cause of p. The collateral circumstances of p change with the connected antecedents of A.

There may, however, be instances where they remain unchanged as represented below :—

II.

(1)	$A\ B\ C$ ..	..	$p\ q\ r$
(2)	$2\ A\ B\ C$ ..	..	$2\ p\ q\ r$
(3)	$3\ A\ B\ C$ ..	..	$3\ p\ q\ r$

In the two sets of instances given above, a consequent and an antecedent rise together. When the antecedent is doubled, the consequent is doubled. The relation between the two may not always be so simple. All that the Canon requires is that the two should vary together according to some definite numerical relation.

The Method of Concomitant Variations may be regarded as an extension of the Method of Agreement in the case of the first set of instances, and as an extension of the Method of Difference in the case of the second set of instances. The inference drawn by this method is therefore subject to the conditions discussed under those two methods.

It is sometimes stated that the Method of Concomitant Variations is a method for determining the quantitative relation between an antecedent and a consequent, when a qualitative relation between them has been established by the Methods of Agreement and Difference. These two methods may prove that a particular antecedent is a cause or a part of a cause of a particular consequent and the Method of Concomitant Variations may then find and prove the quantitative relation between them.

The Method of Concomitant Variations can be applied to determine the qualitative or the quantitative relations between two or more circumstances of a phenomenon, when they cannot be separated from it and when the Method of Difference cannot therefore be applied. In the first four examples given below, the gravity, inertia,

attraction and weight of a body cannot be separated from it and the Method of Difference cannot therefore be applied to determine the effect of any one of these circumstances. In such cases the Method of Concomitant Variations is of great use and is the only method which can be applied to them. It may then be regarded as an extension of the neglected Method of Simple Enumeration by combined observation and experiment.

§ 2. Examples:

(1) The attraction of a material body for another varies directly as its mass; that is, as the mass of a body increases, its attraction for another body also increases: therefore, the antecedent, the mass of a body, and the consequent, its attraction for another, are cause and effect.

(2) The weight of a body varies directly as its mass on the same part of the surface of the earth, that is, as the mass of a body increases, its weight also increases in the same place: the antecedent, the mass of the body, and its weight are cause and effect.

(3) The weight of a body varies inversely as its distance from the centre of the earth. In this case, as the distance of the body increases its weight decreases; and as the distance decreases the weight increases: therefore the weight of a body and its distance from the centre of the earth are causally connected.

(4) The attraction of a body for another varies inversely as its distance from that body; that is, as the distance increases the attraction decreases: therefore the attraction of a body for another and its distance from that body are causally connected.

In these examples, a certain antecedent and a certain consequent vary in a certain manner: in some cases as one increases, the other increases; in some others, as one increases, the other decreases: the inference drawn by the Method of Concomitant Variations is that the antecedent and the consequent are causally connected,—that

the antecedent is the cause or a part of the cause of the consequent.

If the circumstances of the phenomenon of the attraction of a material body for another are regarded as co-existing and therefore incapable of being distinguished as antecedents and consequents, then the circumstances of its mass, its attraction, its weight and its distance from the centre of the earth, varying together in a certain manner, may be the joint effects of a common cause. According to the law of gravitation, two material bodies attract each other inversely as the square of the distance between them. The attribute of being a material body and the power of attraction are regarded as coexistent and cannot be distinguished into cause and effect—it is not known that the one is the cause and the other the effect; but both may be the joint effects of a common cause. The weight of a body is due to the attraction of the earth and varies according to its distance from the centre of the earth. This follows from the law of gravitation which is a generalisation from the phenomena of coexistence of the two attributes, inertia and gravitation, found in all material bodies. The law is established by the Method of Simple Enumeration. It is found by observation that whatever has inertia has the power of gravitation; and a material body is defined by the possession of these two attributes. The mass of a body is defined by its inertia, that is, the resistance it offers to movement. The Method of Concomitant Variations proves that the mass and the power of attraction of a body for another vary together, that the weight of a body varies with the attraction of the earth upon it, and that the latter decreases as the distance of the body increases from the centre of the earth.

(5) A room 10 feet by 8 feet is lighted by one candle. It is then lighted by two candles. It is then lighted by three candles—all of the same size and quality. The differences in the intensity of the lighting of the room

in the three cases are carefully observed by a scientific instrument called Photometer. It is found that the intensity increases with the number of candles lighted. It may, therefore, be inferred by the Method of Concomitant Variations that the number of candles lighted is the cause or part of the cause of the degree of illumination in a particular case. By the Method of Difference, it may be proved that the candle light is the cause of the illumination of the room, by two instances—one in which a candle is lighted and the other in which there is no candle light in the room; and by the Method of Concomitant Variations, a quantitative relation between the quantity of candle light and the degree of illumination of the room may be established.

(6) Heat a certain quantity of water in a vessel by a lamp and observe the rise of its temperature by a thermometer. Heat the same quantity of water in the same vessel for the same period of time by two lamps of the same dimensions and quality as in the first case and repeat the experiment with three lamps of the same dimensions and quality, observing in each case the rise of temperature by a thermometer. The observations show that the temperature increases with the number of lamps, that is, with the increase of heat applied to the vessel. It is, therefore, inferred by the Method of Concomitant Variations that the quantity of heat applied is the cause, or a part of the cause, of the rise of temperature of the water in the vessel. As in example (5), it may be first proved by the Method of Difference that heat is the cause of the rise of temperature; and then a quantitative relation between the heat applied and the degree of rise in the temperature may be established.

(7) The retention of an idea in memory varies with its repetition. It varies also with the degree of attention paid to it. The more an idea is repeated, the greater is its retention. The repetition of an idea is therefore a cause or a part of the cause of the retention of the idea in

memory. The greater the attention paid to the idea each time it is repeated, the greater is its retention. The retention of an idea in memory depends therefore on attention and repetition.

(8) The development of a faculty or function varies with its exercise. The more a faculty is used, the greater is its development. The use or exercise of a faculty is therefore a cause or a part of the cause of its development.

(9) The size and structure of the brain of an animal vary with its intelligence. The greater its intelligence, the larger and the more complex in structure its brain. The intelligence and the brain of an animal are therefore causally connected.

(10) The organisation of an animal varies with its life. The higher the life, the more complex is its organisation. The life of an animal and its organisation are therefore causally connected.

CHAPTER XI.

V. THE METHODS OF RESIDUES.

§ 1. If a part of the consequents of a phenomenon can be shown by deduction from previous inductions to be the effect of some of the antecedents, then the remaining part of the consequents is the effect of the remaining antecedents.

Let $A\ B\ C$ be the antecedents and $p\ q\ r$ the consequents of a phenomenon. If $q\ r$ can be shown by deduction from B and C, to be their effect, then p is the effect of A. The distinction between this Method and the Method of Difference is that in the latter, the part $q\ r$ is shown to be the effect of $B\ C$ by an experiment or observation, while in the Method of Residues, it is deduced from the laws of B and C, established by previous observations and experiments.

(1)	$A\ B\ C$ ..	.. $p\ q\ r$
(2)	$B\ C$ ..	.. $q\ r$

$\therefore$ p is the effect of A.

In the Method of Residues $q\ r$ is deduced from B and C. In the Method of Difference $B\ C \ldots q\ r$ is known by experiment and observation. In the Method of Residues the phenomenon under investigation is the result of an observation or experiment; but the proof that p is the effect of A is deductive, depending on the axiom that if equals be taken from equals, the remainders are equal. It is assumed that there is a sort of equality between the antecedents as the cause and the consequents as the effect and that a part of the ante-

cedents being shown to be equal to a part of the consequents, the remaining consequents are equal to the remaining antecedents.

Mill gives the Canon of the Method as follows :—

"Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents."

§ 2. Examples :

(1) A room 10 ft. by 8 ft. is lighted jointly by two candles of a certain quality and size and a lamp, and the degree of illumination produced by them is observed by a photometer. The degree of illumination of the room, which is produced by a single candle of the same quality and size, has been ascertained by previous experiments. The effect of the two candles can be deduced from the recorded result of the experiments. Subduct this effect from the degree of illumination of the room produced jointly by the lamp and the two candles. Then it may be inferred by the Method of Residues that the remaining degree of illumination is the effect of the remaining antecedent, the lamp. This inference may be verified by an actual experiment with the lamp—that is, the room may be lighted with the lamp only and the degree of illumination produced by it, as observed by a photometer, should agree with the calculated result.

(2) The effect of the heat of a lamp may similarly be determined by heating a certain quantity of water in a vessel by the lamp and two candles, if the heating power of a candle of a certain quality and size has been ascertained by previous experiments.

(3 The existence of cohesion as a separate form of attraction from gravitation can be determined by this Method. The law of gravitation explains many movements of material bodies, but cannot account for the phenomena of the cohesion of the molecules in the same body.

(4) The existence of chemical affinity, vitality and mind as causes is established by this Method. The phenomena of Nature are of different kinds and cannot be accounted for by one and the same cause. Take the chemical phenomena: many of them are due to heat, electricity, light and other physical forces; but there are some, for example, the union of hydrogen and oxygen according to a certain proportion by weight, and of other elements with one another, generalised under the law of chemical combination, that cannot be accounted for by them. It is therefore inferred by this Method that the phenomena which are not the effects of physical forces are caused by a special force called chemical affinity. The phenomena of living things likewise lead to the inference of vitality as a special cause of those phenomena of life which cannot be shown to be the effects of chemical and physical forces. The argument has been applied also to prove the existence of mind as a separate and special cause of those phenomena, for example, of feelings and thoughts of living beings, which are called psychical and which can not be shown to be the effect of vital, chemical or physical forces.

CHAPTER XII.

THE DEDUCTIVE METHOD.

§ 1. Every science consists of facts and phenomena known by observation and experiment and of laws founded on them. The laws are inductions drawn from the phenomena on the ground of the Uniformity of Nature. The inductions are the results of generalisation. They are of different degrees of certainty according to the methods of induction by which they are established. Those that are proved by Mill's methods of experimental inquiry are regarded as more certain than those proved merely by the method of simple enumeration. Those again that are founded on the law of causation are considered to be more certain than those of mere coexistence or of resemblance.

§ 2. The Deductive Method consists in using the inductions of a science as premises for the explanation of the phenomena of the science, for the discovery and proof of the causes, especially, of those phenomena which cannot be investigated by the methods of induction. We have seen that when a phenomenon is the result of a mechanical or homogeneous intermixture of the effects of several antecedent agents or circumstances, the Methods of Agreement and of Difference are ineffective in determining the law of its causation. For the investigation of such a phenomenon, Mill recommends two methods, namely, (1) the Deductive Method, when the individual agents and their laws of action are known by previous observations and experiments, and (2) the Hypothetical Method, when the agents or their laws

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or both are assumed to explain the phenomenon, it being understood that the phenomenon under investigation cannot be explained by any known agents and their laws.

§ 3. The Deductive Method is thus very important in the scientific investigation of phenomena. It consists of three parts:—

(1) The inductions already established in a science.

(2) The calculation of consequences deduced by ratiocination, i. e., by syllogistic and other forms of deductive reasoning, for example, mathematical, from any combination of two or more inductions as premises.

(3) The verification of those consequences by observation and experiment.

The Deductive Method cannot be employed in a science until it has made some progress and established some inductions by observation or experiment or both. If there are no such inductions in it, a science may have recourse to the Hypothetical Method and try to explain its phenomena by assumptions about causes and their laws of action.

§ 4. In Astronomy and Physics there are well-established inductions and the Deductive Method has been very effectively employed in the investigation of the phenomena of those two sciences. The law of universal gravitation is used to explain the phenomena of the celestial bodies. The movements of the planets round the sun and of the satellites around their planets are explained deductively by the law of gravitation. The movements of bodies on the surface of the earth, for example, the phenomena of falling bodies, are explained by the same law. The phenomena of the different seasons of the year are explained deductively by the laws of Physics and Astronomy determining the differences of temperature on the surface of the earth, varying with its position in regard to the sun at different times of the year.

In Physics the laws of reflection and refraction of light, of conduction and radiation of heat, the laws of electricity and magnetism, etc., explain deductively many physical phenomena. All these laws are inductions established by observation and experiment. What is called Mathematical Physics has become possible in virtue of the wide generalisations in Experimental Physics.

The Deductive Method is also employed in Chemistry, Biology, Geology and Psychology as also in Sociology and Politics. In every science there are some laws established by induction from the facts and phenomena known by observation and experiment. These laws may be used as premises for explaining similar phenomena or for predicting future phenomena. They may be used also, as in Geology and Astronomy, for explaining past phenomena. An essential feature of the Method as used in science is that the consequences deduced from the laws must be verified by observation and experiment. It is this character which distinguishes the Deductive Method of Science from the Deductive Method of Geometry. In the latter the results are regarded as true if the premises, namely, definitions, axioms, and postulates are true and if results follow deductively from the premises but the verification of them by observation and experiment is not required.

"To the Deductive Method," says Mill, "thus characterised in its three constituent parts, Induction, Ratiocination, and Verification, the human mind is indebted for its most conspicuous triumphs in the investtigation of nature. To it we owe all the theories by which vast and complicated phenomena are embraced under a few simple laws, which, considered as the laws of those great phenomena, could never have been detected by their direct study." Bk. III, Ch. XI, Sec. 3.

§ 6. The use of the Deductive Method in the investigation of phenomena in science will be illustrated by some familiar examples :—

(i) The phenomena of the movements of air known as winds.

We have the north-east winds in the winter called the trade-winds and the south-west winds in the summer called the monsoons. How are they produced? The Experimental Methods of Enquiry are ineffective in this investigation. We therefore have recourse to the Method of Deduction.

(1) The inductions are those of Physics and Astronomy as regards the effect of heat upon the air, the rise of the lighter air and the movement of the heavier air on the surface of the earth to take its place, the position of the earth in regard to the sun in the winter, and the daily movements of the earth round its own axis, etc.

(2) The consequences drawn from the inductions are that the air in the southern latitudes, being heated by the sun, rises up in the atmosphere and its place is taken by the heavier air from the northern latitudes, causing a north wind and that the direction of this wind is deflected to the east, making it a north-east wind, by the daily rotation of the earth round its axis, as the air in the northern latitudes moves more slowly from west to east round the earth than the air in the southern latitudes on the same side of the Equator in the Northern Hemisphere.

(3) These consequences are verified by the observation of the winds prevailing in the winter in the Northern Hemisphere.

The directions of the winds change when the antecedent agents change their position and the circumstances are thereby altered.

In the summer the northern latitudes are heated by the sun, the air then rises up and its place is taken by the heavier air laden with moisture on the southern latitudes, causing a south wind, and its direction is deflected to the west, making it a south-west wind, by the rotation of the earth round its axis, as the air in the

southern latitudes moves more quickly from west to east round the earth in the southern latitudes than the air in the northern latitudes on the same side of the Equator in the Northern Hemisphere.

These consequences, deduced from the same agents, acting under the changed circumstances but according to their laws, are verified by the observations of the winds in the summer called the monsoons. The rains are due to the vapour with which the winds get charged on the Indian Ocean and the Bay of Bengal: and the vapour is produced by the intense heat of the summer sun on the water of the ocean and the bay. The vapour gets condensed as it comes to the northern latitudes and falls as rain.

The agents producing the phenomena of the north-east and the south-west winds are the sun, the earth, heat, air and water, the circumstances are the movements of the earth round the sun and round its own axis, etc., the laws of the agents are the laws of gravitation, of the expansion of the air and production of vapour by heat, of the movements of the air due to varying pressure, that is, from a region of higher to a region of lower pressure, etc. The phenomena are the results of the mechanical or homogeneous intermixture of the effects of the agents and circumstances concerned.

(ii) The phenomena of falling bodies on the earth.

These phenomena are produced by the action of the earth's gravity upon the bodies and the resistance offered by the air. By the law of gravitation a fruit and a flower detached naturally from a tree would fall to the earth together in the same time; but as they are unequally resisted by the air, the flower being resisted more than the fruit, the former takes more time than the latter to fall to the earth. The amount of resistance offered by the air, being determined by previous experiments and the law of gravitation being known, the movements of the two bodies may be deductively

calculated from them. The calculated movements may then be verified by the observation of their actual movements. If all the circumstances concerned have been taken into consideration, the two should agree.

In the case of projectiles, there is an additional agent, namely, the initial force or impulse with which they are thrown. The effect of this impulse has to be considered in addition to that of gravity and the resistance of the air. The phenomena of the projectiles are the results of the interaction of these three agents and may be deductively calculated from their laws. The calculated results may then be confirmed by actual experiments.

(iii) The phenomena of the formation, by a river, of its bed, its valley and its delta.

From the observations of the actions of rivers at the present time in forming the beds, the valleys and their deltas as they pass through different countries into the sea, laws may be established by induction as to the general nature of their action. These laws may then be applied to explain the formation of the beds, the valleys and the deltas of the old rivers. The agents are glaciers and rainfall, the nature of the rocks, the soil and the configuration of the country through which a river passes into the sea. If the glaciers and rainfall are abundant as in the case of the Ganges or the Brahmaputra, and the country is mountainous and rocky during a part of the course, a river will form deep ravines and wind its way through passes and places offering the least resistance, until it comes to the plains where it opens out and floods the surrounding country, passing through low lands by many mouths into the sea. The flooded parts of the country are renewed year after year and owe their fertility to the alluvium thus deposited on them. The lowlands at the mouth form a delta and rise in level year after year as they recieve deposits of soil from the annual floods. The river will deposit stones and gravel in the mountainous part of its course. These stones

and gravel are rounded and moved by the action of the strong current and bear marks of water-action.

These phenomena which can be deduced from the laws of the agents concerned and the collocation of the circumstances may be verified by actual observation. The phenomena will of course vary according to the collocation of the circumstances and due allowance should be made for variations in different circumstances.

(iv) The phenomena of the union of Hydrogen with Oxygen.

If a certain quantity of hydrogen is mixed with a certain quantity of oxygen and electric sparks are passed through the mixture, the quantity of water that would be formed by their union can be calculated by the Deductive Method, if the law of the combination of hydrogen with oxygen to form water has been previously determined by experiments. It has been ascertained by chemists that two parts by weight of hydrogen unite with sixteen parts by weight of oxygen to form eighteen parts by weight of water. This is the law of their combination. From this law it may be deduced that if the two elements are brought together in certain quantities and made to unite chemically by electric sparks, they will unite with each other according to that law, the excess of either remaining ununited and free. Thus from the quantities given, the quantity of water which will be produced may be calculated and afterwards verified by an experiment. If there is any excess of either element, that is, if there is more oxygen than eight times the weight of the hydrogen in the mixture, some oxygen will remain ununited and free, and if there is more hydrogen than one-eighth of the quantity of oxygen in the mixture, some hydrogen will remain ununited and free. The exact quantity of either in a given case can be calculated and verified by experiment. By the same law may be explained the phenomenon of any quantity of oxygen or hydrogen remaining ununited and free in

an experiment to produce water from a mixture of the two gases, and if the quantity of each gas present in the mixture is known, the quantity of the free gas can be mathematically calculated.

CHAPTER XIII.

THE HYPOTHETICAL METHOD.

§ 1. The tendency to explain phenomena is original and primitive. All science owes its origin to this tendency. Philosophy originated in the tendency to explain the universe. The first European philosopher, Thales, attempted to explain everything by water. He supposed water to be the ultimate cause or agent which by various modifications of rarefaction and condensation had produced all the different things of the world.

He thus adopted the Hypothetical Method. It starts and guides all scientific enquiry. It is as often employed in the daily business of life as in science. It leads on to Induction—to the discovery of scientific truth.

An hypothesis is defined as a supposition made to explain a fact or facts. It admits of different degrees of probability. It is proved by comparing its results or the consequences deduced from it with the fact or facts to be explained. If the consequences agree with the facts the hypothesis is probable. The greater this agreement the greater is the probability of the hypothetical proposition. Its probability increases as it enables us to predict future facts and explain facts already known and generalised under laws. It is thus tested (1) by its adequacy to explain all the facts which it is proposed to explain, (2) by its success in predicting future events belonging to the same class of facts, and (3) by its capacity to explain laws already established by observation and experiment. Even thus tested and proved, an hypothesis does not become a scientific truth or Law of

Nature, until the cause or agent assumed by it is verified, i.e. known to exist by direct perception or by some independent evidence, (2) until the agent is shown to be an adequate cause for all the facts to be explained, and (3) until the hypothesis in question is shown to be the only one tenable and true on the subject, that is, until all other possible hypotheses which may be framed and entertained to explain the same facts are shown to be false or inadequate.

Thus an hypothesis passes as it were through several stages of varying degrees of probability before it becomes an accepted truth of science. At first, it is a mere supposition with hardly any probability—a merely Provisional Hypothesis. It then becomes a Working Hypothesis; attaining a still higher degree of probability it becomes a Legitimate Hypothesis. This is its highest stage as an hypothesis.

From this stage it passes into a scientific truth when it is established by the inductive or experimental methods. If it is established only by the Method of Agreement, it still remains only probable. If it is further confirmed by the Double Method, it becomes more probable. The method of Concomitant Variations may further increase its probability. But until it is proved by the Method of Difference, it does not attain certainty and become an inductive truth. When thus proved, an hypothesis may be called valid or true. An hypothesis has thus the following stages (1) Provisional, (2) Working, (3) Legitimate, (4) Valid or True.

An hypothesis might be also proved deductively by deriving it from one or more well-established Laws of Nature. It would then be a Derivative or Secondary Law of Nature. In this case, also, it would be regarded as certain.

§ 2. An hypothesis may be a supposition (1) about an agent *which is assumed* in order to explain certain phenomena, as the ether in the case of light; (2) about a

collocation or a certain conjunction of known agents, their conjunction or their position in relation to each other being assumed in order to explain certain phenomena, as Copernicus's heliocentric hypothesis; (3) or about a law of an agent, the agent itself being known or sometimes even the agent itself being assumed, as the law of gravitation assumed by Newton to explain movements of the planets, the agent 'force' moving the planets towards the interior of their orbit being a known agent, or as the law of undulation in the case of light, the agent ether being also assumed. In the last case both the agent and its law are assumed.

§ 3. Nature of Hypothesis.

An hypothesis in science must be capable of verification, i.e., of proof or disproof by comparing the results deduced from it with facts or laws.

In the case of an hypothesis about an agent, the existence of the agent must be known by direct perception or by independent evidence, that is, by evidence other than that connected with the facts to be explained.

In the case of a law of a known agent, the agent and the law must be adequate to explain all the phenomena, and all other possible hypotheses must be excluded.

In the case of a collocation also, the above condition must be fulfilled, that is, all other possible collocations must be excluded.

If the above conditions are not fulfilled, the hypothesis cannot be regarded as proved and accepted as a Law of Nature.

The existence of the agent may be incapable of proof. Even when it is known to be a *vera causa*—a truly existing thing, it may not be adequate. And, granting that the agent assumed is real (a *vera causa*) and that it is adequate to explain all the phenomena which it is proposed to explain, even then it is only probable, for there may be other agents which explain the same phenomena. In order that the hypothesis may be accepted as a Law

of Nature, that is, proved as a truth, it must be shown to be the only hypothesis tenable on the subject and capable of explaining the phenomena, all other possible hypotheses being excluded from this claim.

Is this test attainable? Is it possible to exclude all other conceivable hypotheses—conceivable at present or which may be conceived or imagined in future by mankind with the progress of knowledge?

Such exclusion may be possible in Mathematical (e.g. Geometrical) proof.

A is either equal to *B* or less than or greater than *B*. There are here three possible hypotheses—the second and third being excluded, *A* may be inferred to be equal to *B*, and this may be further verified by experiment.

But is such exclusion possible in scientific investigation?

Mill gives an example which, according to him, fulfils this rigorous condition. It is Newton's proof of the central direction of the force which was known to exist as directing the planets towards the interior of their orbit. Newton proved that the direction of the known force must be central, as no other direction explains the phenomenon as generalised under Kepler's second law that the planets describe equal areas in equal times. And he proved that the attraction is inversely as the square of the distance, as no other law would prove the remaining two laws of Kepler.* Mill shows that this proof conforms to the Method of Difference, the only distinction being that the second instance (the negative one) is not obtained by experiment but by deduction.

In this proof Newton assumes the existence of the agent. How is it known? How is it proved to exist?

* The first law of Kepler is that the orbits of the planets are elliptical, and the third law is that the squares of the periodic times of the planets vary as the cubes of the semi-major axes.

This must be either by direct perception or by some independent evidence.

Is the agent (the force assumed) known by Perception? If not, by what independent evidence is it known)?

Is not this force still hypothetical—merely assumed to explain the phenomena and not rigorously proved to be real (a *vera causa*)? For it is not known by direct Perception, nor have we any independent evidence of its existence. The only evidence is the evidence of the phenomena which it is proposed to explain. There must, therefore, be a previous hypothesis about the agent and this hypothesis must be proved by excluding all other possible hypotheses. May not other hypotheses supersede the hypothesis of a force directing the planets towards the interior of their orbits? It has not therefore been proved by the rigorous test which Mill lays down for the conversion of an hypothesis into a Law of Nature.

§ 4. Conditions of a True or Valid Hypothesis, that is, the conditions to which an hypothesis in science *ought* to conform :—

(1) That an hypothesis be verifiable.

This condition requires that the agent assumed must not be supernatural and must be an object of possible experience and that the Law assumed must be sufficiently definite to be proved or disproved inductively or deductively by agreement with facts and by the experimental methods or by derivation from the more general Laws of Nature.

(2) That an hypothesis be adequate.

This condition requires that the agent and its law should fully explain all the phenomena which the hypothesis in question is proposed to explain. If it does not explain all the phenomena, it may require some modification or it may be partly true, i.e., it may explain fully only a portion of the given phenomena or it may be

superseded by some other hypothesis which may adequately explain all the phenomena.

(3) That an hypothesis exclude all other possible hypotheses on the same subject.

This condition requires that the hypothesis in question has been proved to be the only hypothesis which will explain all the phenomena which it is proposed to explain, and that all other possible hypotheses have been shown to be untenable either because they are unverifiable or because they are inadequate.

If an hypothesis does not conform to the above conditions, it is not valid or true. It is not proved to be a Law of Nature. It may be used as a Working or as a Provisional Hypothesis guiding observation and experiment and waiting to be tested by the conditions stated above.

§ 5. Utility of a Provisional Hypothesis in scientific investigation.

When a phenomenon cannot be explained by the known causes of Nature, a new cause may be assumed to explain it; and observation and experiment may be so carried on as to lead to the proof or disproof of its existence.

When a phenomenon cannot be explained by the laws of the known causes of Nature, new laws of the causes may be assumed or the old laws may be modified to explain the phenomenon; and they may be proved or disproved inductively or deductively,—inductively by comparing the consequences which follow from them with the phenomenon to be explained and ultimately by the inductive or experimental methods or deductively by deducing them from more general Laws of Nature. In the earlier stages of proof they suggest and guide experiments by which they themselves are tested and are either proved or disproved. If proved they become more and more probable and are accepted as Legitimate Hypotheses in Science.

§ 6. Working and Legitimate Hypotheses in Science. A working hypothesis in science is one that serves to bring together a number of phenomena by explaining them by some assumed cause or law. It thus concentrates our attention on those phenomena. The phenomena explained by it may be merely a number of instances of the same general phenomenon. It gives a definite direction to the inquiry in respect to those phenomena. It may explain all the phenomena which it is required to explain. It may lead to the prediction of future phenomena of the same class. It may explain empirical laws. When a working hypothesis in science is thus able to explain all the given phenomena, to predict future events, and to subsume under it empirical laws, it becomes what has been called a Legitimate Hypothesis. It is still an Hypothesis—not a Law of Nature or an Inductive Truth. It is only probable, and continues so until it is proved to be the only true and adequate hypothesis. In other words, a legitimate hypothesis is accepted as a Law of Nature only when it has been proved by the experimental methods or deduced from some more general Law of Nature.

In a legitimate hypothesis there may be something still uncertain. The agent for instance may be unknown as in the case of the undulatory hypothesis of light. The law though true may not be adequate as in the case of Darwin's law of Natural Selection. The agent, though known to exist and to be adequate, may not exclude other possible agents as in the case (in my opinion) of Newton's Law of Gravitation. Newton does not exclude the possibility of other agents explaining the laws of Kepler. He establishes the direction and law of action of the force, provided there be such a force. But the laws might be hereafter (and are even at present) explained by other causes. There is the scientific tendency now to do away with force and there was the old tendency to regard the planets as living bodies

(a tendency revived by Fechner). Have we exhaustively proved that no other conceivable hypothesis is tenable? If not, Newton's law of gravitation and his assumption of gravity as an agent are merely Legitimate Hypotheses and cannot be regarded as Laws of Nature or Inductive Truths in Mill's sense.

§ 7. Whewell and Jevons regard all inductions as Legitimate Hypotheses established by the sole condition of their agreement with fact. This condition may be satisfied in three ways:—

(1) An hypothesis may explain all the facts which it is proposed to explain.

(2) An hypothesis may predict future events belonging to the same class of facts which it explains.

(3) An hypothesis may be such that a number of empirical laws—laws established by observation and within a comparatively limited sphere—may be deduced from it. Whewell calls this the "consilience of inductions" and regards it as a great test of the truth of an hypotheses.

Whewell gives these three conditions as the only tests of a Legitimate Hypothesis and does not insist upon any further rigorous proof. Jevons also does not require, in the case of an hypothesis, any proof by the experimental methods. According to him all inductions are at best nothing but well-established hypotheses; and an hypothesis admits of no better proof than agreement with fact. The only proof which it can have is that it explains all the facts, that it is not opposed to any well-established Law of Nature and that it is not self-contradictory, i.e. opposed to the primary Laws of Thought.

CHAPTER XIV.

EXPLANATION.

§ 1. Mill states the sense in which the word explanation is used in science in the following noteworthy paragraph :—

"The word explanation is here used in its philosophical sense. What is called explaining one law of nature by another is but substituting one mystery for another; and does nothing to render the general course of nature other than mysterious. We can no more assign a *why* for the more extensive laws than for the partial ones. The explanation may substitute a mystery which has become familiar, and has grown to *seem* not mysterious, for one which is still strange. And this is the meaning of explanation in common parlance. But the process with which we are concerned often does the very contrary : it resolves a phenomenon with which we are familiar into one of which we previously knew little or nothing; as when the common fact of the fall of heavy bodies was resolved into the tendency of all particles of matter towards one another. It must be kept constantly in view, therefore, that in science, those who speak of explaining any phenomenon mean (or should mean) pointing out not some more familiar, but merely some more general phenomenon, of which it is a partial exemplification; or some laws of causation which produce it by their joint or successive action, and from which, therefore, its conditions may be determined deductively. Every such operation brings us a step nearer towards answering the question which was started in a previous

chapter as comprehending the whole problem of the investigation of nature, viz., what are the fewest assumptions, which being granted, the order of nature as it exists would be the result? What are the fewest general propositions from which all the uniformities existing in nature could be deduced?" (Book III, Ch. XII, Sec. 6, page 549, Vol. I. System of Logic, 8th edition).

In accordance with this meaning, Mill distinguishes three modes of explaining laws of causation, as follows :—

(1) "When the law of an effect of combined causes is resolved into the separate laws of the causes, together with the fact of their combination."

(2) "When the law which connects any two links, not proximate, in a chain of causation, is resolved into the laws which connect each with the intermediate links."

(3) In the first and second modes, one law is resolved into two or more; in the third, two or more laws are resolved into one, that is, subsumed under a more general law.

§ 2. Examples of the first mode of explanation.

(1) The law of the phenomena of the north-east or trade winds is resolved into the laws of the agents concerned and their collocation as explained in Chapter VIII on the Deductive Method.

(2) The law of the phenomena of the monsoons is resolved into the laws of the same agents with their changed collocation as explained in Chapter VIII.

(3) In the same chapter is also explained the law of falling bodies on the earth as deduced from the law of universal gravitation and the law of resistance offered by the air. The law of projectiles is deduced from those two laws and the law of mechanical action.

(4) In vision the law of the formation of an image on the retina is resolved into the laws of the transmission of the rays of light from a luminous object through the air, the lens, and other media of the eye.

(5) The rise of mercury in a thermometer when its

bulb is brought in contact with a hot body is explained by the laws of conduction of heat and of expansion of mercury by heat. In the case of a clinical thermometer used by a physician to determine the temperature of a patient, its bulb is kept for some time in contact with a part of the body so that the heat of the blood may pass into the mercury and both may be of the same temperature.

(6) The rise of mercury in a barometer is explained by the laws of the weight of the atmospheric air, of the transmission of pressure in fluids and of the equality of pressure in fluids at the same level. The rise is higher the greater the weight of the column of the atmospheric air pressing upon the mercury. The weight and the pressure exerted by it upon the mercury vary with the temperature of the air and the moisture present in it.

§ 8. Examples of the second mode of explanation:

(1) In the example given of the phenomenon of lighting a match, the rubbing is popularly regarded as the cause of the flame; but between this cause and the effect, there are many intermediate links, the supply of which would be an explanation of the law of causation of the flame by rubbing. First there is the mechanical law of causation of heat by friction. Then there is the law of the union of the chemicals under the influence of heat. There are the further laws of the production of heat by chemical union and of the conversion of the chemical products into gases by heat and the kindling of the stick by the hot gases.

(2) In the production of a sensation of light by a luminous object, the object is popularly regarded as the cause of the sensation; but between the object and the sensation there are many intermediate links, some of which are physical, determined by the laws of optics, some physiological, determined by the laws of physiology, and some psychological, determined by the laws of mind. The image or change on the retina of the eye is explained

by the laws of physics, the transmission of the image or the change produced by it through the optic nerve to the brain is explained by the laws of physiology, and the consciousness of the affection thus produced, which is called sensation, is dependent on the laws of mind, especially the laws of sensibility and attention. A sensation would have for its proximate cause attention directed to a sensuous affection of the mind. If the affection is strong the attention is almost spontaneous and instinctive. If the attention is otherwise engaged, the sensuous affection would not be followed by the consciousness of a sensation. The change in the retina and the physiological processes through the nerve and the brain are also indispensable conditions and links in the whole process of the production of a sensation of light. So are also the physical processes of the transmission of rays of light from the luminous object through the air and the different media of the eye.

(3) A seed is planted in the soil and it germinates into a little plant consisting of a root growing downwards into the earth and of a stem rising upwards. Between the germ in the seed and the little plant there are many processes going on in the seed and outside under the influence of the heat and light of the atmosphere and the moisture and other ingredients of the soil. The tracing of these processes would be an explanation of the germination of a seed.

(4) "Food nourishes": food is popularly regarded as the cause of the nourishing of the body. Out of the ingredients of the food are gradually built up the different tissues and organs of an animal; but there are many processes through which the food passes before the different ingredients can be taken up by the different parts of the organism. There are mechanical, physical, chemical and vital processes, which form the subject matter of physiology, and the tracing of these would lead to the finding of the laws of causation in the building up and nourish-

ing of the body. This is true also in the case of a plant which has its food partly from the soil and partly from the atmosphere; and out of the ingredients of this food are formed the roots, the trunk, the branches, the leaves, the flowers and the fruits by processes which form the subject matter of the physiology of plants.

§ 4. Examples of the third mode of explanation.

(1) The laws of expansion of liquids, of solids, and of gases by heat are subsumed under the general law of expansion of material bodies by heat.

(2) The laws of Heat as a mode of motion, of Light as a mode of motion, of Electricity as a mode of motion, of Mechanical action as a mode of motion, are subsumed under the general law of the conservation of energy as a mode of motion. It is found by experiment that if a certain quantity of heat disappears in a particular situation, a certain quantity of mechanical movement takes its place, thus showing that the two quantities—one of heat and the other of mechanical movement—are equivalent to each other, as both are really different modes of motion. Such equivalence in their effects has been proved by experiment to exist between other forms of energy; and the laws of the different forms of energy have been subsumed under the general law of the conservation of energy.

(3) The different laws of perception in the several senses of touch, sight and hearing have been subsumed under the general law of perception of an external object. This law is that in every perception, there is a sensation which is referred to an external object as its exciting cause. In tactual perception there is a sensation of touch, which is referred to an external tangible object as its exciting cause. In visual perception there is a sensation of sight which is referred to an external visible object as its exciting cause. In auditory perception there is a sensation of hearing, which is referred to an external sounding object as its exciting cause. From these laws in the three special

senses is generalised the law of perception which is applicable to all the senses. In perception by smelling and tasting there are also sensations which are referred to external objects as their exciting causes.

(4) The different laws of reproduction of mental phenomena are subsumed by some psychologists under the general law of contiguity, and by some others under the general law of similarity, resemblance or likeness, and by some others under the two laws.

(5) The law of falling bodies and the law of attraction of the planets towards the sun and of the satellites towards their planets, have been subsumed under the law of universal gravitation. This law is that all material bodies attract each other directly as their mass and inversely as the square of their distance. By this law are explained the movements of the planets round the sun, of the satellites round their planet, and of bodies falling on the earth. In the case of the planets and satellites it is assumed that they have a motion of their own along the tangent of their present orbits, their existing movements being the resultant of this motion and the central attraction due to universal gravitation.

§ 5. In all these cases and modes of explanation a partial law is referred to a more extensive law; two or more less general laws to a more general law. One phenomenon is first classed with another phenomenon resembling it, and the two are brought together under a law. Other phenomena resembling the two are referred to the same class and explained by the same law. Partial laws thus established are subsumed under a more general law; and the process goes on until the most general laws are discovered. Mill draws a distinction between (1) derivative laws which are deducible from, and may, in any of the modes stated above, be resolved into other and more general ones, and (2) ultimate laws which cannot be thus deduced or resolved. He adds:—"We are not sure that any of the uniformities with which we are acquainted

are ultimate laws; but we know that there must be ultimate laws; and that every resolution of a derivative law into more general laws, brings us nearer to them" (Book III, Ch. XIV, Sec. 1). Are there any limits, asks Mill, to this operation or may it proceed until all the uniform sequences in nature are resolved into some one universal law? Mill holds "that the ultimate Laws of Nature cannot possibly be less numerous than the distinguishable sensations or other feelings of our nature," i.e., those which are distinguishable from one another in quality, and not merely in quantity or degree. Phenomena differing in quality must have different causes and cannot be brought under the same law. Colour, for example, is a phenomenon *sui generis*. Sound is another. Each must be referred to a different cause, having its own law. "The ideal limit, therefore," says Mill, "of the explanation of natural phenomena would be to show that each distinguishable variety of sensations or other states of consciousness, has only one sort of cause." He points out that phenomena differing in degree may have their complex laws resolved into laws of greater simplicity and generality. This has been successfully done in the case of the phenomena of motion, which are produced in various ways but which differ in degree only. Differences of duration, or of velocity, are evidently differences in degree only; and differences of direction in space are no differences at all as they disappear by a change in our own position. In the case of the other distinguishable states of consciousness differing in quality, for example, moral and aesthetic consciousness, there are ultimate laws which cannot be deduced from, or resolved into, higher laws. It should be noticed that whether a mental state differs from another mental state in quality or merely in quantity and degree must be discerned by consciousness that is, directly known by intuition, or immediate perception.

§ 6. In regard to the Laws of Nature, Mill remarks:

"The laws, thus explained or resolved, are sometimes said to be accounted for, but the expression is incorrect, if taken to mean anything more than what has already been stated. In minds not habituated to accurate thinking, there is often a confused notion that the general laws are the causes of the partial ones; that the law of general gravitation, for example, causes the phenomenon of the fall of bodies to the earth. But to assert this would be misuse of the word cause: terrestrial gravity is not an effect of general gravitation but a case of it, that is, one kind of the particular instances in which that general law obtains. To account for a Law of Nature means, and can mean, nothing more than to assign other laws more general together with collocations, which being supposed, the partial law follows without any additional supposition" (Bk. III, Ch. XII, Sec. 6).

The laws, whether ultimate or derivative, are not the causes of the phenomena of Nature. They presuppose "a number of permanent causes, which have subsisted ever since the human race has been in existence, and for an indefinite and probably an enormous length of time previous. The sun, the earth, and planets, with their various constituents, air, water, and other distinguishable substances, whether simple or compound, of which nature is made up, are such Permanent Causes. These have existed, and the effects or consequences which they were fitted to produce have taken place (as often as the other conditions of the production met) from the very beginning of our experience. But we can give no account of the origin of the Permanent Causes themselves. Why these particular natural agents existed originally and no others, or why they are commingled in such and such proportions, and distributed in such and such a manner throughout space, is a question we cannot answer" (Bk. III, Ch. V, Sec. 8). It is true that we cannot answer this question by the methods of science. Scientific explanation reaches its upward limits when it

has traced the existing phenomena of Nature to their Primeval Causes and discovered the laws of these causes. The original distribution of the causes and their nature and properties cannot be explained by any methods of science. "The coexistence, therefore," says Mill, "of Primeval Causes ranks, to us, among merely casual occurrences." But the question asked by Mill is a persistent one, and has not been left unanswered by the human mind. It implies a notion of cause quite different from that which Mill has adopted for the purposes of Inductive Logic.

CHAPTER XV.

MILL'S DOCTRINE OF CAUSE.

§ 1. Mill says at the very outset of his inquiry into the Law of Causation (Bk. III, Ch. V, Sec. 2) that he does not propose to enter into the metaphysical question of "the origin and analysis of our idea of causation," that when he speaks of the cause of any phenomenon, he does not mean a cause which is not itself a phenomenon, that he makes "no search into the ultimate or ontological cause of anything," and that the causes with which he concerns himself are not *efficient*, but *physical causes*.

"The only notion of a cause," says Mill, "which the theory of induction requires, is such a notion as can be gained from experience. The Law of Causation, the recognition of which is the main pillar of inductive science, is but the familiar truth, that invariability of succession is found by observation to obtain between every fact in nature and some other fact which has preceded; independently of all considerations respecting the ultimate mode of production of phenomena, and of every other question regarding the nature of 'things in themselves.'" (Section 2).

Mill seeks the physical cause and not the efficient cause of a fact. Every fact which has a beginning has a cause and this cause is another fact. "For every event," says Mill, "there exists some combination of objects, or events, some given occurrence of circumstances, positive and negative, the occurrence of which is always followed by that phenomenon" (Section 2). The antecedent fact, or combination of objects or events, is called the cause, and the consequent event or phenomenon the effect.

There is a tendency to call the *proximate* antecedent event rather than any of the antecedent states or permanent facts cause of the phenomenon, but this is not always the case; sometimes any one of the antecedent states or conditions, positive or negative, is called cause of the phenomenon. Mill recognises this popular view of cause, but sets against it what he calls the philosophic, that is, the scientific view and defines cause as "the sum total of the conditions, positive and negative taken together: the whole of the contingencies of every description, which being realised, the consequent invariably follows" (Section 3). The negative conditions of the event are summed up under one head, namely, the absence of preventing or counteracting causes.

The sequence between the antecedent and the consequent must be not only invariable but also unconditional, that is, the consequent must not depend on any other condition. If it does, that condition must be included in the cause.

"We may define, therefore," concludes Mill, "the cause of a phenomenon, to be the antecedent, or the concurrence of antecedents, on which it is invariably and *unconditionally* consequent. Or if we adopt the convenient modification of the meaning of the word Cause, which confines it to the assemblage of positive conditions without the negative, then instead of 'unconditionally,' we must say, 'subject to no other than negative conditions.' (Bk. III, Ch. V, Sec. 6.)

§ 2. Various objections have been taken to this view of Cause. Comte and Whewell object to it on the ground that the word cause should not be applied to "events." They insist on its usage in the sense of efficient cause. Comte considers the inquiry into causes as vain and futile, while Whewell recognises the investigation of causes as part of science. Martineau objects to Mill's doctrine of Cause on the ground that the notion of cause implies not only efficiency as insisted upon by Comte and

Whewell, but also the power of making the indeterminate determinate, of choosing between alternative courses, and holds that the notion of Cause is derived from, and is identical with, the notion of Will as directly given in our voluntary actions. He further points out that sequence is not the essence of causation, and that the "unconditionality" required by Mill cannot be proved in the case of any physical antecedent.

§ 3. To all these objections Mill firmly replies that it is necessary to have a term for the invariable and unconditional antecedent of a phenomenon; that the term cause should be retained and used in science in this sense; that he is concerned only with physical causes; that of "the efficient causes of phenomena, or whether any such causes exist at all," he is "not called upon to give an opinion"; that with Will as an ontological cause, he has no concern, but that he recognises volition as a mental fact; that science treats of facts and phenomena and is concerned only with physical or phenomenal causes; and that for the purposes of Inductive Logic it is not necessary to inquire into "the ultimate or ontological cause of anything" or into "the ultimate mode of production of phenomena."

Mill thus gives a new meaning to the word cause against the protests of the contemporary philosophers and logicians and makes the Law of Causation the foundation of the Canons of Induction. His view has prevailed and the term cause has come to mean in Science and Inductive Logic physical or phenomenal Cause as defined by him. In Religion and Ethics, it still means Cause in the ontological sense. A full discussion of the subject belongs to Metaphysics.

NOTE.

(*a*) Some recent critics of Mill (Green and Professor Bernard Bosanquet) have pointed out that "unconditionality" as a character of cause would embrace the whole universe as the cause of every event, as in their opinion, all events are interconnected and are parts of the whole of Nature. The universe is, according to them, a self-determining unity consisting of inter-dependent parts. They regard the Unity rather than the Uniformity of Nature as the ultimate ground of induction and derive the Law of Causation from the Law of Sufficient Reason, entirely overlooking the fact that Mill gives a new meaning to the word cause in accordance with the use of it in Science and regards the law of causation as an induction or a valid generalisation from our experience of the facts and events of Nature. We have seen that Mill excludes from his causal inquiry the metaphysical question of the ultimate origin of the universe and is concerned only with the practical problem of determining the relations and connections of events by the methods of experimental science.

The Unity of Nature as conceived by Green and Professor Bernard Bosanquet implies that all the phenomena of nature belong to one and the same Subject (or Self) that necessarily determines them. It does not necessarily imply that there are identical or similar elements, or that there are universal relations, or that, under the same conditions, the same effect follows. These conceptions are contained in Mill's idea of Uniformity. The idea of the Unity of Nature is compatible with the idea of an *immanent* Subject which is fully expressed in nature as well as with the idea of a *transcendent* Subject which is only partially expressed, and which may therefore introduce from within itself new elements that may interfere with the course of Nature. In the latter case there would be unity without uniformity.

The Uniformity of Nature as held by Mill is not an instinctive, intuitive or metaphysical principle but an induction or a valid generalisation from the facts and phenomena of nature, confirmed by all our experience.

(*b*) To the objection that an induction involves the fallacy of *petitio principii*—because every induction depends on the Uniformity of Nature and the Law of Causation, both of which are established by induction, induction being thus proved by induction—Mill replies as follows :—

"The assertion, that our inductive processes assume the law of causation, while the law of causation is itself a case of induction is a paradox, only on the old theory of reasoning, which supposes the universal truth, or major premise, in a ratiocination, to be the real proof of the particular truths which are ostensively inferred from it. According to the doctrine maintained in the present treatise, the major premiss is not the proof of the conclusion, but is itself proved, along with the conclusion from the same evidence. 'All men are mortal' is not the proof that Lord Palmerston is mortal; but our past experience of mortality authorizes us to infer both the general truth and the particular fact, and the one with exactly the same degree of

assurance as the other. The mortality of Lord Palmerston is not an inference from the mortality of all men, but from the experience which proves the mortality of all men; and is a correct inference from experience, if that general truth is so too." (Book III, chapter XXI, section 4. Mill admits that the principle of the Uniformity of Nature or the Law of Causation may be regarded as the major premiss of a syllogism to which an induction may be reduced; but shows that both the major premiss and the conclusion of the syllogism are proved by the same evidence of collective fact, viz., the facts of observation and experiment justifying the universal proposition as well as the particular one. All the past and the present cases of the death of men constitute the evidence or proof from which is inferred the universal proposition "all men are mortal" or the particular one "Some man, John Smith, now living will die" or the proposition "All the kings now living will die." All the past and present cases of the uniformity of nature and causation are the evidence or proof of the principle of the uniformity of nature and the law of universal causation, as also of any particular case of uniformity and of causation in future. From the past cases of causation of phenomena, a universal law may be inferred and then applied to a particular phenomenon or class of phenomena under investigation in the form of a syllogism, but the evidence for the truth of the universal proposition and of the particular or less general one is the same, viz., the collective fact of the causation of past and present phenomena.

The old theory of reasoning, referred to in the above passage, is the traditional theory which treats Logic from Aristotle's point of view and proceeds throughout on his theory of Universals. Mill treats Logic from the modern scientific point of view and regards experience as the basis of all universal synthetic propositions, including the law of causation, the principle of uniformity of nature and the axioms of geometry. The traditional theory considers a universal proposition a necessary element of a legitimate process of inference, while Mill holds that the particular instances known in the past and present form or constitute the evidence of the truth of a universal proposition. Thus the law of causation is grounded, according to him, not on any more general principle, but on the instances past and present, in which it has been exemplified and also verified from time to time in our experience. He maintains the same view in regard to the principle of the uniformity of nature which is wider than the law of causation, as it embraces the laws of coexistence and of resemblance, besides those of succession.

It is by experience that Mill finds Nature to be orderly, intelligible and rational while the traditional theory assumes it to be so on *a priori* grounds. This is the fundamental difference between Mill and those logicians (e.g. among recent writers Mr. H. W. B. Joseph, Fellow and Tutor of New College, Oxford, in "An Introduction to Logic," 1906), who follow Aristotle and adopt the traditional theory. The difference is due to their different points of view—one being scientific and the other metaphysical.

(*c*) Some followers of Mill (Bain and Professor Carveth Read) regard the conservation of force as an essential factor of Causation and con-

sider that, unless there is transfer of force from the cause to the effect, there is no causation. They define force as matter in motion and take the hypothesis of the conservation of force to mean really the conservation of motion. Mill points out that the definition of force as matter in motion does not include the force which may be in matter at rest. The force, for example, in the coal which is afterwards manifested as heat when it combines with oxygen, is not actually existing as motion in the molecules of the coal. It is a property of the coal, which, under certain conditions, produces motion which is called heat. "The true definition," says Mill, "of force must be not motion, but potentiality of motion. What the doctrine, if established, amounts to is, not that there is at all times the same quantity of actual motion in the universe, but that the potentialities of motion are limited to a definite quantity, which cannot be added to, but which can be exhausted; and that all actual motion which takes place in Nature is a draft upon this limited stock. It needs not all of it to have ever existed as actual motion. There is a vast amount of potential motion in the universe in the form of gravitation which it would be a great abuse of hypothesis to suppose to have been stored up by the expenditure of an equal amount of actual motion in some former state of the universe. Nor does the motion produced by gravity take place, so far as we know, at the expense of any other motion." (Book III, Chap. V, Sec. 10, p. 406 of Vol. I, 8th Edition.)

In a wider sense force may be defined as potentiality of change, i.e., capacity to produce a change under certain conditions. Every phenomenon, mental or physical, is a change. A mental phenomenon is a change in time, while a physical phenomenon is a change in both time and space, that is, it is a motion. The conservation of force in this wider sense or even in Mill's sense as potentiality of motion has not been established. As the word "force" implies something hidden and metaphysical, physicists have replaced it by the term "energy," and the law at first known as that of the conservation of force has been designated the law of the conservation of energy. This law is true in the case of an isolated material system if no addition is made to its energy from within or without. It would not be true if there were a source within, such as is implied by Mill's idea of force as potentiality or property which under certain conditions produces motion.

(*d*) Bain and Professor Carveth Read distinguish among the antecedents of an event an inciting power and a collocation of circumstances or position of objects. In regard to this distinction, Mill points out (Book III, Chap. v, Sec. 10, p. 407) that as the cause of the event consists of objects and circumstances, that, as the objects have forces in the form of properties, the so-called inciting power is included in his definition of cause, and that any special mention of it is not only unnecessary but would be tautological.

(*e*) By adopting Bain's theory of an inciting power and a collocation of circumstances or position of objects among the antecedents constituting the cause of an event and by defining force as matter in motion, Professor Carveth Read has been obliged to exclude all but material phenomena from the category of Causation. There is no causation unless there is an inciting power or force; and there is no force unless there is

matter in motion; therefore, there is no causation unless there is matter in motion. The exclusion of mental phenomena from the relation of cause and effect is not compatible with Mill's law of universal causation. It is opposed to his practice of applying the law to mental phenomena; and it is not a legitimate deduction from the law.

CHAPTER XVI.

FALLACIES.

§ 1. A fallacy in the widest sense is an error of any kind. It arises from the transgression of the rules of the processes described in the preceding chapters. An orderly statement of the fallacies or errors which may arise is given below :—

I. The fallacies or errors of Observation.
II. The fallacies or errors of Classification.
III. The fallacies or errors of Definition.
IV. The errors of Terminology and Nomenclature.
V. The errors of the Hypothetical Method.
VI. The fallacies or errors of inductive inference arising from the transgression of the rules of inference by the different methods of induction :—

(1) The Method of Simple Enumeration.
(2) The Method of Agreement.
(3) The Joint Method.
(4) The Method of Difference.
(5) The Method of Concomitant Variations.
(6) The Method of Residues.

VII. The fallacies or errors of Analogical Inference.
VIII. The fallacies or errors of the Deductive Method.
IX. The fallacies or errors of Explanation.

Besides the above, there are fallacies or errors of Intuition and Immediate Perception, the treatment of which belongs to Psychology and Metaphysics.

§ 2. The fallacies of observation are distinguished into those of (1) non-observation and (2) mal-observation, according as they arise from the overlooking of some instances of a class or from the wrong observation of some instances. This often happens when the

instances overlooked or wrongly observed are likely to go against our pre-conceived opinion. If we are favourably impressed by a person, we are likely to overlook those instances of his action which would go against our opinion of him. In the contrary case when we are badly impressed by a person we are likely to mistake even his good actions for bad. In science when we have already formed an induction, we are apt to overlook instances which go against it. In Botany we may overlook instances which are exceptions to the class as defined by us; though in virtue of their general qualities they may claim to belong to the class, thus causing some modification in our definition of the class. Again in observing and describing the various parts of a plant, certain parts may be overlooked or there may be error in actual observation due to imperfect development of the faculty or to incorrect inference from what is directly perceived.

§ 3. The errors of Classification arise in mistaking the superficial qualities of things for their fundamental attributes. The errors are mainly due to the imperfect knowledge of the things classified. They are generally eliminated with the progress of our knowledge of those things. This has been the case with our classifications of almost all things, especially of plants and animals. Unscientific classifications lead to fallacious inferences. For example, if the bats are classified with the birds because they can fly, there will be many inferences drawn in regard to the bats, which will not be true of them. The other characters common to the birds will be ascribed to the bats; and it will be found on observation that the bats do not possess them. The bats have no feather which is the most characteristic quality of the birds. The wings of the bats consist of membranes, spread out over the elongated fingers of the fore-limbs and continued over those of the hind-limbs, while the wings of the birds consist of long quill feathers attached to their fore-limbs.

§ 4. The errors of Definition are also due to our imperfect knowledge of the things defined. The definition of a scientific term is relative to our knowledge of the things denoted by the term. Every definition is more or less incomplete. Moreover a thing or a class of things may be defined for a particular purpose in view of a special science. Such a definition may be regarded as sufficient for that science; but necessarily incomplete from a wider point of view. For example, water may be defined in Physics as a liquid with a certain specific gravity; in Chemistry as a compound of hydrogen and oxygen, i.e., as the monoxide of hydrogen; in Geology as a great denuding agent; in Meteorology as the vapour present in the air, and so forth. These definitions of water refer to its different qualities, but do not exhaust them. A complete definition of water is an ideal which has not been yet realized in science. It is much more difficult to define living things. The definition of a plant or an animal is difficult, because they grow and have different attributes at different stages of their development. The definition of animals as a class is still more difficult, because what is usually attempted is to find out what is common to all animals from the lowest to the highest. Such a definition is not of much use. It becomes too abstract for a clear conception of the thing defined. There seems to be some uncertainty as to the method that should be adopted and the subject of definition has not therefore attracted much attention of the Inductive Logician. Mill himself regards all definitions as nominal, that is, as mere statements of the meanings, popular or scientific, of the terms defined. For the faults of Definition in this sense, see my "Text-book of Deductive Logic," Part I, Chap. II, Sec. 7.

To show the error or fallacy involved in taking nominal definitions for scientific, we may take the example of the scholastic definition of man as a rational animal.

It is a definition of man *per genus et differentiam* in the Aristotelian fashion. That is, the definition is a statement of the genus animal to which man belongs and of the differentia rational by which man is distinguished from other species, that is, other animals belonging to the same genus. This definition of man involves the fallacy that the other animals are irrational, i.e., devoid of Reason and that all men are possessed of it. Are all other animals devoid of Reason? What is Reason? Are all men possessed of it? The question, What is Reason, is a difficult one and would be answered differently by philosophers of different schools. Animals have intelligence and cannot be said to be entirely devoid of Reason. Hence, some philosophers prefer to call them not irrational but sub-human beings. Until Reason is defined we can not say whether all men have it or not. If Reason comprehends both Speculative and Practical Reason, it is doubtful whether all men are possessed of it. On the other hand, if it is reduced to mere reasoning and prudence, it is shared by animals with man. The scholastic definition of man is a remnant of the philosophy of Aristotle.

§ 5. The fallacies of Terminology and Nomenclature arise from the ambiguity and shortage of terms in a language. In a perfect language every word should have a definite meaning and every meaning, that is, everything, whether of the internal or the external world, should have a word signifying it. This is very far from being the case in any language. The result is that the same word is often used in different senses; and a language has many ambiguous terms. The confusion that arises from the existence of ambiguous terms in a language is nowhere more evident than in Metaphysics and Psychology. There is hardly a philosophical term which has not more than one meaning. As the object of the study of Logic is to acquire the habit of accurate and clear thinking, it is very desirable that the student

should be fully aware of the existence of ambiguous terms and make an attempt at the very outset to have clear conceptions of the different meanings of a term. Fallacies of course arise when a term used in one sense is taken in another sense. An ambiguous term is really equivalent to two terms if it has two meanings, as is recognised in Deductive Logic. Unfortunately a term has sometimes more than two meanings; and it is a good exercise on Terms in Deductive Logic to distinguish the different meanings of a term and to describe its logical characters in each of these senses.

In Botany and Zoology it is necessary to have a sufficient number of terms for describing the organs and the parts of the organs with their modifications of a plant or an animal. If these terms were ambiguous, it would be impossible to identify a plant or an animal from their description. In Descriptive Botany, there is, therefore, a large number of words simply for accurately describing the different plants. In Comparative Anatomy there is also a large number of words for distinguishing and describing accurately the different bones, nerves, muscles, ligaments, etc., of the various classes of animals. In Systematic Botany and Zoology there are words or names for the different classes and each class-name stands for a larger or a smaller group of individuals or groups of individuals. The terms kingdom and sub-kingdom, class and sub-class, order and sub-order, genus and sub-genus, species and sub-species, variety and sub-variety, race and family indicate varying groups of plants or animals, each requiring a distinct and unambiguous name.

§ 6. In the case of the Hypothetical Method, the errors are due to the overlooking of the conditions to which a valid hypothesis ought to conform. These conditions have been fully discussed in the Chapter on the Method.

In testing an hypothesis, the points to be especially noticed are:

(1) Whether the agent, if the hypothesis assumes one, is real, that is, whether its existence has been verified, i.e., known directly by intuition, perception, observation or experiment.

(2) Whether the law of the agent explains the phenomena for which the hypothesis is framed;—that is, whether the consequences deduced from the law agree with the phenomena to be explained.

(3) Whether the hypothesis explains all the facts and phenomena for which it has been framed. It often happens that some residuary phenomena cannot be explained and the hypothesis in question may require some modification.

(4) Whether the agent and its law are compatible with the already known agents and their well-established laws.

(5) Whether there is any rival hypothesis claiming to explain the same phenomena. If there be any, the question of settling which one has the greatest claim becomes an important one. The claims of two rival hypotheses are sometimes settled by what has been called an *Experimentum crucis*, that is, an experiment (or observation) which settles between their rival claims and points out one as false and the other as true or at least more probable than the other. This is the case when a consequence drawn from the one is falsified, while a consequence drawn from the other is verified, by a particular experiment.

Exercise.—Test the following Hypotheses:—

(1) The rotation of the earth produces days and nights.

(2) The vapour of the air produces dew.

(3) The revolution of the earth produces periodic changes called the seasons.

(4) The mind is a mirror and the perceptions are the images of the external objects.

(5) Heat is not a material substance but a mode of motion.

§ 7. The Errors of Inference by the different Methods of Induction :

The rules and conditions of correct inference by the different Methods of Induction have been fully discussed in the Chapters devoted to these methods. In this Section attention will be drawn only to the fallacies or errors which usually occur.

Under the Method of Simple Enumeration comes the error of inferring the antecedent to be the cause of the consequent. This method cannot establish the relation of cause and effect. The antecedent inferred as cause consists usually of more or fewer circumstances than those necessary to produce the effect. The induction itself may be disproved by any contrary instance discovered at any moment.

Under the Method of Agreement comes the error of inferring an antecedent to be the only cause of the consequent, i.e., the error of overlooking the fact of plurality of causes, and also the error of regarding an antecedent as alone capable of producing an effect when the antecedent can produce it only along with one or more other antecedents.

Under the Method of Difference comes the error of regarding the antecedent as more than an indispensable part of the cause of the phenomenon.

Under the Joint Method is often committed the mistake of regarding the conclusion as more than probable. The conclusion of this method as also of the Method of Agreement is not as certain as that obtained by the Method of Difference.

Under the Method of Concomitant Variations comes the error of regarding the two varying circumstances as cause and effect; for they may be the joint effects of one and the same cause or they may be mere concomitants without any causal relation.

Under the Method of Residues there may be errors in the previous Inductions or in the process of Deduc-

tion as involved in the act of assigning part of the consequents to part of the antecedents.

§ 8. Analogical Inference varies in probability according to the degree of resemblance between the two things. Errors arise when superficial resemblance is mistaken as fundamental and the inference is considered to be more probable than it really is.

In Inductive Logic there are no methods for determining quantitatively the degree of probability according to the degree of resemblance. We have to be satisfied with such vague terms as "probable," "very probable," "most probable," etc., expressing the degrees of probability. Where a causal relation can be established between the common properties of the two things, and the attribute inferred of the one because it is possessed by the other, the analogical inference attains the highest degree of probability, i.e., certainty, and gives rise to an Induction. In Book II, Chap. III, Sec. 7, Mill recognises analogical reasoning as the type of all reasoning and shows that an analogical inference may be resolved into an induction plus a syllogistic application of the induction to a particular case. When an analogical inference does not lead to an induction but gives rise only to a probable proposition, the latter may be used as a premise in a probable syllogism. See my "Text-book of Deductive Logic," Part III, Chapter IX, Probable Reasoning and Probability.

§ 9. In the case of the Deductive Method there may be errors in the establishment of the Inductions, there may be fallacies in the process of Ratiocination, and lastly there may be errors of Observation and Experiment in Verification.

We have already treated of the errors of Observation and of Induction. The fallacies of Ratiocination are treated at great length in Deductive Logic.

§ 10. Explanation in two of its three forms is either the deduction of phenomena from certain well-established

laws or the subsumption of certain laws under a higher and more general law. The errors in these two cases will, therefore, be either those of the Deductive Method or those of the Hypothetical Method. Explanation in the remaining form is the discovery and proof of the connecting links between a cause and an effect. The errors in this case will therefore be those of observation and experiment and of induction.

Mill points out that some people make the serious mistake of regarding laws as causes, and I may add that some people make even a more serious mistake in regarding physical causes as ultimate or ontological. The inquiry into physical causation forms an endless chain. There is no limit to it from a scientific point of view. The cause or causes reached by the methods of science are themselves effects and not ultimate causes. Some scientific men, however, assuming the rôle of the metaphysician, set up their terminal (i.e. terminal for them) cause whether it be ethereal, atomic, dynamic, or vital, as ultimate and ontological, entirely overlooking the very pertinent question asked by Mill in regard to his permanent and primeval causes, namely, how have their number, their properties and their distribution in space been determined? This question can be answered only from a metaphysical view of cause—from the notion of a cause which being itself uncaused has determined the number, the properties and the distribution of the physical causes. Such a cause is not physical and phenomenal, but spiritual and ontological with freedom to choose and power to create according to his choice. A full discussion of this subject belongs to Religion and Ethics.

APPENDIX.

THE DEFINITION AND PROVINCE OF LOGIC.

§ 1. Mill's Definition of Logic.*

(1) Logic defined as "the science of Proof or Evidence." (Vol. I, section 4, para. 8). "Evidence in the proper sense of the word" is distinguished from "the evidence of consciousness."

What is meant by "the evidence of consciousness"? and what is meant by "evidence in the proper sense of the word?"

"The evidence of consciousness" is the assurance that comes from one's own consciousness of a fact—from the perception or direct observation of a phenomenon. By this sort of evidence, intuitive truths are known. "Perception, Memory, Intuition, Consciousness, Belief (primary as distinguished from inferential) bear evidence to facts or phenomena revealed by them. This sort of evidence is distinguished by Mill from "evidence in the proper sense of the word," in which one phenomenon bears evidence to the existence of another phenomenon. The second sort of evidence is inferential evidence. When one fact is inferred from another fact, the latter is the evidence of the former. Mill regards this sort of evidence as the proper subject of the Science of Logic.

Is "evidence" identical with "proof"? Is there any distinction between the two?

If by "proof" is meant "demonstration," it is of two sorts—(i) Inferential, (ii) Experimental. One can demonstrate or prove a fact by an experiment—by actually producing it before an audience. This is experimental

* The references are to Mill's Logic, 2 Vols., 8th Edition, 1872. The subsequent editions are reprints of it.

proof. And a fact or phenomenon can be demonstrated by showing how it follows from fundamental principles or from assumptions, as the phenomenon of reflection of light can be proved by the Undulatory Theory.

Is evidence also of these two sorts,—(i) Inferential and (ii) Experimental?

If so, (ii) will correspond to what Mill calls "the evidence of consciousness." When a fact is known directly or immediately by perception, no further evidence is necessary. Perception then is a form of evidence. So is the consciousness "I am hungry." This is known directly by consciousness and here the evidence of consciousness is sufficient to establish it as a fact. No reasoning or inference is wanted. But such evidence, according to Mill, is not a subject-matter of Logic. Whether a particular fact is revealed by consciousness or not, whether it is known by perception or not, whether it is intuitive or not, whether memory bears testimony to it or not, are questions lying beyond the jurisdiction of Logic. They belong, according to Mill, to Metaphysics. Logic is concerned with that portion of our knowledge and belief, which can be justified by reasoning. But this reasoning cannot be indefinitely carried on. It must ultimately rest on data supplied by Consciousness, Intuition, Perception, Memory, etc.

(2) Logic defined as "the science of the operations of the Understanding which are subservient to the estimation of evidence: both the process itself of advancing from known truths to unknown, and all other intellectual operations in so far as auxiliary to this." (Section 7, para 1, p. 11.)

These auxiliary operations are those of—

(1) Naming,
(2) Definition,
(3) Classification.

Logic analyses reasoning with the object of distin-

guishing correct from incorrect reasoning. It need not carry the analysis further than is required for this purpose.

Metaphysics analyses reasoning to its ultimate elements and attempts to distinguish reasoning from intuition.

§ 2. Province of Logic.

"The province of Logic," says Mill, "must be restricted to that portion of our knowledge which consists of inferences from truths previously known; whether those antecedent data be general propositions or particular observations and perceptions." (P. 8, section 4, para. 8.)

Mill says:—

"Truths are known to us in two ways—some are known directly, and of themselves; some through the medium of other truths. The former are the subject of Intuition, or Consciousness; the latter, of Inference. The truths known by intuition are the original premises from which all others are inferred. Our assent to the conclusion being grounded on the truth of the premises, we never could arrive at any knowledge by reasoning, unless something could be known antecedently to all reasoning." (P. 5, section 4, para. 2.)

As examples of truths known by immediate consciousness, Mill gives: "I was vexed yesterday", "I am hungry to-day" and says that all our bodily sensations and mental feelings belong to this class.

As examples of truths which we know only by way of inference, Mill refers to "the events recorded in history," "the theories of geometry", or "the occurrences which took place while we were absent," etc.

"Whatever we are capable of knowing", says Mill, "must belong to the one class or to the other; must be in the number of the primitive data, or of the conclusions which can be drawn from these."

With the first class Logic has nothing to do. "Whatever is known to us by consciousness, is known beyond

possibility of question. What one sees or feels, whether bodily or mentally, one cannot but be sure that one sees or feels. No science is required for the purpose of establishing such truths; no rules of art can render our knowledge of them more certain than it is in itself. There is no logic for this portion of our knowledge." (Page 6, section 4, para. 5.)

"But," says Mill, "we may fancy that we see or feel what we in reality infer. A truth, or supposed truth, which is really the result of a very rapid inference, may seem to be apprehended intuitively." Such is our perception of distance by the eye which seems so like intuition but is, in reality, an inference grounded on experience.

How to distinguish these apparent intuitions from real ones?

How to distinguish the facts which are the objects of intuition or consciousness from those which we merely infer?

"But this inquiry," says Mill, "has never been considered a portion of Logic. Its place is in another and a perfectly distinct department of science, to which the name Metaphysics more particularly belongs: that portion of mental philosophy which attempts to determine what part of the furniture of the mind belongs to it originally, and what part is constructed out of materials furnished to it from without." (Cf. Kant and Reid) [p. 7, section 4, para. 7.]

To Metaphysics belong the questions of:

(1) The existence of Matter.
(2) ,, ,, Spirit.
(3) The distinction between Spirit and Matter.
(4) The reality of Time and Space and the inquiries into the nature of—
   (*a*) Conception,
   (*b*) Perception,
   (*c*) Memory,
   (*d*) Belief.

"To this science must also be referred the following, and all analogous questions: To what extent our intellectual faculties and our emotions are innate, to what extent the result of association. Whether God, and Duty, are realities, the existence of which is manifest to us *a priori* by the constitution of our rational faculty; or whether our ideas of them are acquired notions, the origin of which we are able to trace and explain; and the reality of the objects themselves a question not of consciousness or intuition, but of evidence and reasoning." (Section 4, para. 7, p. 8.)

§ 3. Definitions of Logic rejected by Mill:—

(1) As the "Art of Reasoning." (Section 2, p. 2.)

(2) As "the Science, as well as the Art, of Reasoning." Whately—Ditto.

Reasoning has been generally taken by writers on Logic to mean "Syllogising." It has not been taken as it ought to be in the wider sense to mean "to infer any assertion from assertions already admitted": and in this sense induction is as much entitled to be called reasoning as the demonstrations of geometry. Whately's definition is rejected on the ground that it is too narrow—that it excludes induction and confines Logic to Syllogistic Reasoning.

(3) As "the Art of Thinking", "Port Royal Logic." (Section 3, p. 4.)

(4) As "the science which treats of the operations of the human understanding in the pursuit of truth." (Section 3, p. 4.)

This definition is rejected on the ground that it is too wide—that it includes such operations as Conception, Perception, Memory, and Belief, all of which are operations of the understanding in the pursuit of truth, but which, according to Mill, belong to Metaphysics and not to Logic.

According to Mill, there is no Logic of Observation.

"Logic," says Mill, "neither observes, nor invents, nor discovers; but judges" (Section 5, p. 9). "Logic is the common judge and arbiter of all particular investigations. It does not undertake to find evidence, but to determine whether it has been found (ditto)." Logic is "*ars artium*;" the science of science itself. All science consists of data and conclusions from those data, of *proofs* and what they prove: now, Logic points out what relations must subsist between data and whatever can be concluded from them, between proof and anything which it can prove." Ditto.

(5) As "the Science of the Formal Laws of Thought." (Section 7, p. 14, footnote.)

This definition is given by the Hamiltonian School. It is rejected on the ground that it is too narrow. It "restricts the science to that very limited portion of its total province, which has reference to the conditions, not of Truth, but of Consistency (ditto)." It excludes, "as irrelevant to Logic, whatever relates to Belief and Disbelief, or to the pursuit of Truth as such." (Ditto).

§ 4. Relation of Logic to Metaphysics and the special sciences.

Metaphysics is the science of intuitive truths and Logic of inferential truths. Metaphysics treats of immediate knowledge and Logic of mediate knowledge.

Whatever is known by Intuition or Consciousness comes under Metaphysics, and whatever is known by inference from truths previously known comes under the jurisdiction of Logic.

Are there any truths known by Intuition? Yes, whatever is known by Consciousness is intuitive—e.g., "I am hungry."

Intuitive truths may be general propositions or particular observations and perceptions. Mill discards the first class and restricts all intuitions to the second class, viz. "particular observations and perceptions."

According to Mill all general propositions are generalisations from particular observations and perceptions and come, therefore, under the authority of Logic, while all particular observations and perceptions are independent of it. Logic has nothing to do with the truth or falsity of a *particular* fact in a special science. Logic is not a science of Observation but of Reasoning. Given certain particular facts known by observation, Logic judges what may be inferred from them. The facts themselves are given by sense-perception and their truth or falsity cannot be determined by Logic. If the so-called facts of sense-perception are really inferences, they come under the authority of Logic; but if they are directly or immediately revealed by our "consciousness," they come under Metaphysics.

Now what does Mill mean by "consciousness" and how are we to distinguish what is directly given by consciousness from what is inferred from it?

Here Mill would reply that this enquiry does not belong to Logic but to Metaphysics. Mill has, however, in different parts of his work, taken it up and given his own answer to it.

Intuitionists hold that consciousness reveals not only particular facts but general principles. The axioms of geometry, the fundamental principles of science, the laws of thought, etc., are, according to them, intuitive, i.e. directly and immediately known by consciousness. They, therefore, exclude them from the jurisdiction of Logic and refer them to Metaphysics. They define Metaphysics as the science of first principles and distinguish it from the special sciences as treating of the particular facts known by Sense-Perception and they define Logic as a practical science which on the basis of certain first principles lays down the conditions and tests to which the particular facts must be submitted in order that they may give rise to a science.

According to Mill, there are no general principles

known intuitively. These are all the results of inference from particular facts which alone are given to us by Consciousness.

Mill discusses this question under the head of "Demonstration and Necessary Truths" and decides it against the Intuitionists and the Kantians.

Granted that particular facts alone are known to us by Consciousness or Intuition, what is the nature of these "particular facts"? And what is the nature of "Consciousness" and "Intuition"?

Is there any growth in consciousness? Does a child's consciousness reveal the same facts as the consciousness of an adult? What determines the growth of consciousness? How much is it subject to education, expansion and development in respect to any particular sense? Logic would have nothing to say against facts made directly known to us by consciousness, nor can Logic give an account of the process of growth of consciousness. For the process is not one of inference from given data; but of expansion, and development, like that of a plant from its seed, from the first manifestation of consciousness. Mill would say that the inquiry into the nature of this process belongs to Psychology and Metaphysics and not to Logic. Logic is concerned only with the correct processes of inference and not with the processes of growth and development of consciousness in man. In the present state of scientific inquiry, we should say that Psychology undertakes to give an account of the latter processes as Biology gives an account of the processes of the growth and development of life. Given the matured human consciousness with all the particular facts, simple or compound, directly known to us, Logic can determine certain relations among them. On the basis of the facts, Logic can determine the relations of sequence, coexistence and resemblance among them, and when such relations have been determined and made the basis of general propositions, these can be applied

to particular cases. The knowledge of the particular facts is, therefore, entirely excluded from Logic; and the determination of the relations of facts in general and their application to particular cases belong to Logic.

Logic is, therefore, concerned with the processes of reasoning adopted by all the special sciences—with what is common to all of them; and Metaphysics comes under the authority of Logic in so far as it adopts the processes of Reasoning. Metaphysics has thus a twofold relation to Logic. In one respect, as treating of intuitive truths or of the facts of consciousness, Metaphysics is independent of Logic. But as adopting the processes of reasoning (and if it is a science it must reason), it comes under Logic which treats of all forms of reasoning. Metaphysics, in so far as it is a reasoned discourse or science, must conform to the laws and rules of Logic.

All the special sciences in so far as they adopt reasoning and its connected processes in order to discover the relations of sequence, coexistence and resemblance among their respective phenomena, come under the jurisdiction of Logic.

Logic as a formal science does not treat of any particular facts or any particular department of phenomena but of those conditions and tests of reasoning and its connected operations, to which all facts must conform in order that they may constitute a science. Logic is thus distinguished from the special sciences such as Physics and Chemistry which begin with facts known to us by observation and experiment and then proceed, on the basis of these facts, to construct by logical methods a superstructure of a body of inferential truths.

§ 5. Distinction between Logic and the special sciences.

A special science has recourse to observation and experiment to find out the facts. These facts are then subjected to certain processes such as Analysis and

Synthesis, Classification, Definition, Hypothesis, Generalisation, Induction, Deduction, etc., in order to discover their relations of sequence, coexistence and resemblance. These relations, when discovered, become the laws of those facts. For example, the science of Chemistry consists of the chemical facts and the laws to which the facts conform.

Logic treats of those conditions to which all the special sciences must conform when they apply to their respective groups of facts, the processes of Analysis and Synthesis, of Classification and Definition, of Hypothesis and Explanation, of Induction and Deduction. It does not in the first place find the particular facts of any department. It does not, therefore, have recourse to observation and experiment to determine the truth or falsity of the data of any special science.

It does not in the second place determine the relation between one particular fact and another in a special department of phenomena. This is also done by the special science to which the facts belong in accordance with the rules of the Logic of that special science. There is thus a distinction between a special Logic (or an applied Logic) of a particular science and the general Logic of all the special sciences. The latter treats of those conditions which are universally applicable to facts and phenomena of all the special sciences. These are conditions to which Analysis and Synthesis, Definition and Classification, Induction and Deduction, Hypothesis and Explanation in all the special sciences must conform, and Logic treats of these universal conditions of the various processes of correct thinking employed in all the sciences. The rules of Logic are applicable not only to Theoretic or Positive sciences, but to such Practical sciences or Arts as Politics and Ethics, and also to the daily business of the magistrate, the military commander, the navigator, the physician, the agriculturist, etc. They are applicable even to the science of Metaphysics in so far

as it has recourse to reasoning and its subsidiary processes. Thus Metaphysics, also, like the special sciences, comes under the jurisdiction of Logic. It is independent of Logic, in so far as it treats of intuitive truths or of the ultimate data of knowledge directly given to us by Consciousness or Intuition. The special sciences also are independent of Logic in so far as their data are concerned. The data of Chemistry, for instance, are known by observation and experiment and they are as independent of Logic as the facts of consciousness referred by Mill to Metaphysics. Thus Logic has nothing to do with any particular facts, whether given by Sense-perception, or by Consciousness and Intuition.

§ 6. Dr. Venn's view of Logic.

"Logic is not an ultimate science." P. 3, "Empirical or Inductive Logic. 1889."

"Logic then as here conceived is neither a purely objective nor a purely subjective science. It involves elements, consisting essentially in the relation of one to the other, and serious error results from the neglect of either aspect, and even from insufficient recognition of it." P. 22.

Dr. Venn maintains that "a system of comprehensive Logic must postulate, must in fact take as its basis, a fundamental duality." "This twofold aspect," says Dr. Venn, "of the science—objective and subjective—is so important a characteristic that it will be perpetually presenting itself in various applications throughout this work. It seems to me almost peculiar to Logic amongst the sciences. There are some, like Psychology, in which the primary reference is throughout to the mental processes, and there are others, like the ordinary physical sciences, in which the primary reference is throughout to the external phenomena. But a science like Logic, which has to do with the processes of the human mind when judging about phenomena, and, more particularly, with the processes of gradu-

ally extending our knowledge of those phenomena, occupies necessarily an intermediate position. The treatment here adopted may, indeed by comparison, be called Material or Objective,—I have chosen to insist here and elsewhere upon the convenience of this designation of my conception of Logic,—but it must be remembered that the epithet is employed to mark the departure from the extreme subjectiveness of the customary treatment." (pp. 26-27.)

Logic, like Mathematics, is an "applied" or "hypothetical" science. The assumptions made as to the simplicity and abstract perfection of the materials of the Logician are not strictly true like the corresponding assumptions in Mathematics. (See p. 39.)

Logic is a science; but intentionally or unintentionally, actually or hypothetically, Logicians lay down also rules for practice. Logic has thus been treated both as a science and as an art. "The strict province," says Dr. Venn, "of the Inductive Logician, as has been so often insisted upon already, is that of science pure and simple. But uniform and consistent adherence to this province is out of the question. Our science is meant to be put in practice." Page 565.

"Logic, investigating all the various sciences, makes abstraction of what may be called the forms of them, that is, of certain elements which are common to them all. By such abstraction we obtain certain processes, such as those of induction and deduction, or the more elementary processes into which these may admit of being analysed; and certain materials or results, such as terms, propositions, definitions and so forth. The elements thus common to all the sciences and special to none, make up what we call Logic; at any rate they comprise the bulk of what is here understood by Material or Inductive Logic." (P. 588.)

Dr. Venn thinks that the forms of, or the elements and processes common to, all the various arts may be

abstracted from them and treated under what may be called the Art of Arts corresponding to the name of Logic as the science of sciences. The Art of Arts would thus treat of the general Rules and Process of Action of all the various Arts, abstracted from what is special to each Art.

The Art of Arts would lay down Rules to which our conduct in all departments must conform. It would thus be general Art of Action. Such questions are now discussed only in connection with Ethics. But it is evident that they arise also in connection with Political Economy, Jurisprudence, Statistics, Sociology, etc. All these have a practical side and determine our conduct. So far as these are principles of action (of conduct or practice) common to all these practical sciences, they may be generalised and treated under the Art of Arts.

It seems that the ancient division of Philosophy into (1) Theoretical and (2) Practical was founded upon considerations to which Dr. Venn draws attention. Theoretical Philosophy treated of what is and Practical Philosophy of what ought to be with reference to certain ends to be achieved. The former gave rise to Science and the latter to Art. The former treated of the knowledge of the various kinds of beings or things as they are; and Logic was the science of the sciences of the various classes of things and treated of the forms or processes and elements common to them all. The latter treated of the action of man in various departments with reference to the ends to be attained; and the principles (or rules and processes common to all kinds of action or to action in all departments) were studied under a General Art founded upon Practical Philosophy.

Practical Philosophy would treat of the various ends of human action and determine their relation to one another. It would be the science of Conduct or Practice in the widest sense of the term. It would co-ordinate the various practical sciences and determine their pro-

cesses in relation to one another. It would thus embrace all of them as Theoretical Philosophy embraces all the sciences of what is. And as Logic treats of the forms of the latter, General Art would treat of the forms of the former. Logic treats of the form of Knowing. General Art would treat of the form of Action. As Logic analyses knowing, Art would analyse Acting or Conduct in the widest sense.

The term Art might therefore be revived in its old sense and applied to this new formal study and might be distinguished into general and special. General Art corresponding to General Logic would treat of what is common to all the various practical sciences or science of conduct, and a special Art corresponding to the Logic of the special sciences would treat of a particular practical science. The term Art might be generalized and used to mean what is defined here as General Art; a special Art might be described with reference to the practical science with which it is connected, as in the case of the Logic of a particular science.

§ 7. Criticism of Mill's View of Logic.

Mill's view may be criticised (1) from the Hamiltonian or (2) from the Hegelian point of view. It is also liable to attack from the scientific side and from Ueberweg's point of view. Let us take these one by one:—

(1) Criticism from the Hamiltonian point of view.

Mill is unable, says the Hamiltonian School, to justify his conclusion from the known to the unknown, from some to all. By no logical principle, can we pass from some to all. This is opposed to a fundamental logical rule of immediate inference—proposition "*A*" cannot be inferred from "*I*."

According to the Hamiltonian School, the conclusion must be contained in the data. Nothing can be inferred in the conclusion, which is not implicit in thought in the premisses. Mill violates this fundamental logical rule and fails to justify his conclusion.

Mill appeals to the law of the Uniformity of Nature, especially to the law of Causation. But how is this law proved? Where is the guarantee of the universal truth of this law? At best it is only a presumption, an hypothesis proved by the agreement of inferences drawn according to it, with facts known by observation and experiment. It does not therefore attain to the certainty and universality of a Logical Principle like the fundamental Laws of Thought which are necessary. It is only a probable law of phenomena and has therefore no place in Logic which is the science of the formal Laws of Thought.

Mill's inferences and reasonings, says the Hamiltonian School, are psychological rather than logical. They are determined by the psychological laws such as those of Association. Mill fails to distinguish the Psychology of Reasoning from its Logic.

Mill appeals to experience for the justification of his process and laws of Reasoning. As experience verifies the conclusion, the laws and the processes must be regarded as correct. The Hamiltonian School replies that Logic has nothing to do with experience. Logic treats only of the Forms of Thought, i.e., of the forms of Concepts, Judgments and Reasonings, and not of its objects. Logic has nothing to do with the empirical truth or falsity of its data. It is concerned only with their formal truth. Its concepts, judgments and reasonings ought to be realisable in thought. They may or may not be realised in experience, and Logic has nothing to do with experience.

If Logic were dependent upon any Law of Nature, known by experience, it would lose its present certainty and necessity and universality. For the law on which it was based might change with experience, might turn out to be false, and might be superseded by another law. But Logic treats of principles which cannot be changed and must always remain true of all objects of thought. Such principles must be *a priori* and not *a posteriori*, in-

tuitive and not experimental—due to the nature of the thinking subject and not to the object known which may be different in different provinces of Nature or in distant parts of the Universe.

§ 8. (2) Criticism of Mill's view from the scientific side.

Mill's Logic aims at real or scientific truth. How can real truth be attained by inference, if the data given by perception and pure observation are not true, accurate and exhaustive? And how can Mill be sure of the truth and accuracy of his data or of their being exhaustive in the subject under investigation, unless he includes in his Logic a part devoted to Perception or Observation and Experiment, Consciousness or Intuition? Mill must bring the perceptive or intuitive as well as the inferential truths under the jurisdiction of Logic. He must treat of Intuition, Perception, Observation and Experiment as well as of Induction and Deduction, etc.

Mill is, however, persistent in repeating that Logic treats only of a certain portion of our knowledge, namely, that large portion which is the result of the process of reasoning (a process of knowing employed in daily business as well as in science). According to him Metaphysics treats of the contents of Consciousness or Intuition and the various special sciences give us the particular facts made directly known to us by observation. Logic takes these as its data and applies to them the general forms or rules of Induction and Deduction, Classification and Division, Analysis and Synthesis, Hypothesis and Explanation and thus transforms them into a science. Logic is thus a formal science, treating of the forms or general principles or common features and elements of the various processes by which the particular facts of all the special sciences are transformed into a coherent body of knowledge. Mill holds that there are such general principles or forms and that his Logic gives an account of them.

But the question is, are not the conclusions drawn from the data according to the general principles of Mill's Logic true only so far as the data are true, accurate and exhaustive? . . . . .

If there is any error in them, or if they are not exhaustive, the conclusions would not be really true, so that even in Mill's Logic, the conclusion is only hypothetically true.* Mill would, I think, admit this and regard logical principles and rules hypothetical in the same sense in which mathematical principles and rules are. Dr. Venn points out that Logic is like Mathematics "Applied", or "Hypothetical" (p. 39, "Emp. Logic").

What Mill means or ought to mean by the inference being really true is that, given certain facts as data, Logic lays down rules to which one must conform in order that we may pass from these facts to others which are *connected with* them but not *contained* in them or *implicit* in them. Given the effect, the cause may be inferred. Given that A is the invariable antecedent of B, then when B is known by observation, A may be indirectly known by inference and verified afterwards by experiment. Mill says that facts in nature are connected with one another and that it is possible to determine their connections and that these connections are constant and regular (and not arbitrary and irregular). The object of Inductive Logic is to prove these laws on the basis of the facts known by observation and experiment and also to help the scientific man as far as possible to discover them, but not to go beyond, or to ascertain the truth or falsity of, the particular observations and facts, which must be done by the special science to which they belong.

§ 9. (3) Criticism from Ueberweg's point of view.

Ueberweg includes Perception under Logic and devotes a part to it. He defines Logic as "the science of the

* *Cf.* my "Text-book of Deductive Logic." Pt. III, ch. viii, sec. 5—The hypothetically necessary character of all Deductive Inference.

Regulative Laws of Human Knowledge." His Logic treats of the criteria of all truth.

Ueberweg would object to Mill's exclusion of Perception and Consciousness from Logic. He would point out that material or real truth cannot be made the end of Logic unless the truths known by Perception and Self-consciousness are brought within the jurisdiction of Logic. Logic in short cannot realise its end—the attainment of truth—unless it treats of all the processes of knowing and lays down conditions to which they must conform in order that they may attain their end, namely, truth. The form of Perception like the form of Reasoning *should* be treated in Logic. The form of Perception is what is common to the processes of Perception as employed in the various special sciences. Logic should separate this common feature of Perception and lay down conditions to which Perception in general as employed in all the sciences should conform in order that it may be valid. Logic should investigate the processes by which objects are directly or immediately made known to us as well as the processes by which they are indirectly or mediately known. Every science starts with truths directly known and proceeds either deductively (as in Mathematics) or inductively (as in Physics and Chemistry). The general principles in the former case and the particular facts in the latter case are known immediately and are different in the various special sciences; but the processes by which they are known have certain common features or elements. These common features should be investigated by Logic as a science of the Regulative Laws of Human Knowledge. When discovered they would furnish the criteria by which the validity of immediate knowledge would be determined.

Thus scientific men and the school of Ueberweg insist alike on the inclusion of Perception or Observation and Experiment in Logic. But there is a difference between them in their insistence. Scientific men would identify

Logic with the sciences and make the data of observation and experiment as well as the processes of thinking the subject-matter of Logic. According to them, the form cannot be separated from the matter of knowledge, and Logic cannot be treated apart from science. If Logic is to attain its end, namely, material or real truth (or truth as known in experience) as defined by the schools of Mill and Ueberweg, it must comprehend all the sciences, it must be identified with the science of the Universe and the various special sciences must be regarded as its various branches. Logic would then become identical with Philosophy as the science of the Universe and would correspond, on the one hand, to Hegel's conception of it as identical with the science of Being (thought and being are identical and Logic and Metaphysics treating respectively of thought and being are also identical) and, on the other, to Spencer's conception of Logic as the science of the most universal relations and correlations of things. But as Spencer has nowhere fully developed his conception of Logic as a science, it is difficult to say how he would establish it without examining the various data of the special sciences and how he would distinguish it from his conception of Philosophy as the science of the Universe.*

Ueberweg would not identify Logic with Metaphysics as Hegel does, nor would he identify it with Philosophy as the science of the Universe embracing the special sciences as its various branches. He would distinguish the form of perception and the forms of the other kinds of knowledge from their matter or content and treat of the former in Logic and of the latter in a special science. He would thus separate Logic from Metaphysics and the special sciences. With Ueberweg, Logic is a regulative science like Ethics and Aesthetics, and is a branch of

* On Spencer's View of Logic as an Objective Science, see my "Text-book of Deductive Logic," Appendix E, Section 3.

the Philosophy of Spirit. He gives the following scheme of the sciences (English Translation of Ueberweg's Logic by Thomas M. Lindsay, M.A., 1871. Section 6) :—

System of Philosophy.

I. Metaphysics (science of principles common to nature and spirit).

II. Philosophy of Nature (science of impersonal existences)—comprising the physical and natural sciences.

III. Philosophy of Spirit (science of minds or personal and self-conscious existences) comprising :

(i) Psychology.

(ii) Logic or science of the laws of truth.

(iii) Ethics or science of the laws of goodness.

(iv) Æsthetics or science of the laws of beauty.

(v) Pedagogy.

(vi) Philosophy of History.

§ 10. (4) Criticism of Mill, Ueberweg and Hamilton from the Hegelian point of view.

All these Logicians draw a distinction between the form and the matter of thought and knowledge. The form is the subject of Logic and the matter the subject of the special sciences. The form of knowledge is the same in all the sciences, while the matter of knowledge varies and gives rise to the distinction between one science and another.

The matter of knowledge in the case of external phenomena comes from the not-self. It consists of the sensations of the different senses. The form of knowledge comes from the faculty of knowing, according to Ueberweg, and from what is common to all the scientific processes, according to Mill.

Hegel abolishes the distinction between the form and the matter and ascribes both to the same source, namely,

the thinking and active Reason or Self. According to him, the Self produces both the form and the matter of all thought and knowledge. There is no thought without both form and matter. There is no form apart from the matter and there is no matter apart from the form. The form and the matter of thoughts are relative as well as correlative to one another. The matter of a thought passes into the form of a second thought and this form again becomes the matter of another thought. Logic is the science of the Organism of Thought and traces its development from the lowest to the highest stage—from the category of Pure Being to the notion of the Absolute Idea. The Organism of Thought develops according to the Law of Thesis, Antithesis and Synthesis and gives rise to the various categories. It starts from Being, and passes through the stages of Quality, Quantity, Degree, Identity, Ground, Substance, Cause, Reciprocity, Mechanism, Chemism, Teleology, Life and Cognition, and it attains its highest stage in Self-Consciousness. With Hegel, Logic is the most comprehensive science comprising all other sciences and is identical with Metaphysics and Theology.

§ 11. Thought.

Thought may be treated—

(1) Psychologically,

(2) Logically, and

(3) Metaphysically;

and the Laws of Thought are different according to these three aspects of Thought.

(1) Psychologically, the enquiry is into the origin and development of thought in the child and in the adult, in the individual and in the race. The Laws of Thought, from this point of view, are the laws of succession, co-existence and resemblance among the various forms or modifications of Thought.

§ 12. (2) Logically, the inquiry is into the validity (i.e. truth or falsity) of thought, and the Laws of Thought

are the conditions to which thought must conform in order that it may be valid, i.e., agree with reality.

Logically, thought implies a standard by which it is to be measured and its value for knowledge determined.

Thought corresponding to, or agreeing with, Reality is usually called knowledge; and Logic thus becomes the science of the Laws regulative of Knowledge. (Cf. Ueberweg's definition of Logic).

Reality may be regarded as something absolute—existing independently of mind and therefore also of thought—and the end of logical thinking may be regarded as the attainment of agreement or correspondence with Reality. Ordinary everyday thinking may or may not correspond with reality; it may correspond partly or not at all; and there is much difference in the thinking and the thoughts between one individual and another, between the child and the adult, between the educated and the uneducated, between the savage and the civilised. Psychology treats of these differences and accounts for them. Logic has a standard to which all thought must come up. Logic has an end which must be satisfied by thought. The end is the agreement with reality. Logic lays down the conditions to which thought must conform in order that it may agree with reality and attain its end. The logical value of thought is to be determined by this criterion (or test).

§ 13. According to some Logicians (Hamilton, Mansel), the end of Logic is not agreement with reality as defined above but truth in the sense of what can be realized in Thought according to the three fundamental Laws of Identity, Contradiction and Excluded Middle. They consider these Laws to be the Laws of Logical Thought, and regard any thought conforming to them as logically valid. They start with fully developed percepts, each consisting of a large number of attributes given by perception which according to them does not

come under Logic but belongs to Psychology and Metaphysics, the former treating of its subjective aspect as a mental process and the latter of its objective aspect, as a form of knowledge having reference to reality.

From these given percepts, they obtain concepts by Analysis, Comparison, Abstraction, Generalisation, etc., according to the Laws of Thought. What is common to a large number of (individual) percepts is regarded as a concept which may thus consist of a single attribute or of an aggregate of attributes, separated by thought from the percepts and formed into a sort of unity by thought aided by language.

From the percepts and the concepts are obtained judgments according to the Laws of Thought. All judgments are, according to them, analytical, the predicate being identical with the whole or a part of the subject.

From the judgments given as premisses are obtained inferences according to the Laws of Thought. All the inferences are, according to them, propositions which are implicitly contained in the premisses Nothing can be inferred which is not in some form present in the premisses.

In Immediate Inference there is only one premiss, and each of the inferences follows from it, according to the Laws of Thought.

In Mediate Inference there are two or more premisses and the conclusion follows from them jointly and is implicitly contained in them.

All inferences and processes in Logic are necessarily true, i.e., follow necessarily according to the Laws of Thought from the given percepts. According to them Logic has nothing to do with experience or with truths given to us by experience, by perception or by self-consciousness, by intellectual consciousness, by æsthetic consciousness, by moral consciousness, or by religious consciousness. The truths or facts revealed to us or made directly known to us by these various forms of con-

sciousness, intuition, perception, or pure observation are beyond the scope and jurisdiction of Logic. They form the subject-matter of the various special sciences such as Physics and Psychology, and must be known under special conditions of perception and consciousness, which are treated in these special sciences respectively. Logic could pronounce no opinion on the truth or falsity of the data—the facts made known to us directly—of these sciences. Given the facts as existing in adult consciousness and sense-perception, Logic can formulate the conceptions and judgments and draw inferences, according to the Laws of Thought.

§ 14. Here may be noted the view of Mill, who would agree with the fundamental position stated above (that truths revealed by Consciousness are beyond the jurisdiction of Logic) and contend at the same time that the end of Logic is truth or reality as known in Experience and not merely as realized in Thought. He maintains that from the ultimate data as revealed by consciousness or pure observation, external and internal and forming the subject-matter of the various special sciences, Logic draws inferences which agree or ought to agree with facts which may be known in experience, and that Logic as a Science analyses correct reasonings and finds out their essential conditions and as an Art lays down the rules to which reasoning must conform in order that it may lead to conclusions which agree with facts as known in experience. All the various special sciences subject their facts or data, given to each of them by pure observation or consciousness, to certain processes, in order to transform them into a body of coherent truths, as a system of knowledge or a science. These processes have certain common features or characters, which may be separated from what is peculiar to each special science and made the subject-matter of a science. Logic is the science which treats of these common characters or features (i.e. the forms) of the processes employed by all the special

sciences. Hence Logic is the science of sciences. Hence Logic is the art of arts. It treats of the conditions to which all reasoned knowledge must conform and to which therefore a large portion of the knowledge embodied in every science—that portion, namely, which is the result of reasoning or inference (and in every science a large portion is obtained in this way)—must conform.

§ 15. According to Idealists reality is not something absolute, existing independently of mind. It is something dependent on a mind, finite or infinite.

(1) According to the Subjective or Empirical Idealists, all reality is phenomenal and dependent on the subject or the Ego. The real is the phenomenal. What I am conscious of, is real to me. That of which two or more minds are alike conscious is real to those minds. That of which all human minds are alike conscious is regarded as real, i.e., objectively real, true, existing; and this is regarded as the object of knowledge, the end of logical thought.

(2) According to the Absolute Idealists, reality is dependent on the absolute mind and exists in relation to this mind. We know it so far as we have in us the nature of the absolute mind. Thinking must be so conducted as to lead to thought as it exists in the Absolute Mind. Thought as existing in the Absolute Mind is truth, and when the latter is known by man it is called knowledge.

§ 16. Logic, from the Absolute Idealistic point of view, treats of the conditions to which our thought must conform in order that it may agree with Truth, i.e., with thought as existing in the Absolute or Divine Mind; and from the Empirical or Subjective Idealistic point of view, it treats of the conditions to which our thought must conform in order that it may agree with relative human thought, i.e., with thought as it exists in the most cultured minds at a particular age of the world.

It is evident that the latter is progressive. The

thought of the cultured minds varies with the state of culture in a country as well as with the age of the world. It varies with the progress of Science and Philosophy. There is no finality to the progress and development of thought in the human mind. The greater the experience of the world, the fuller will be the thought of mankind. Logic would thus vary with culture, with the progress of Philosophy and Science; and the conditions of correct thinking at one age may not entirely agree with those at a remote age. They must be in harmony with our theory of the Universe, of the World or Nature, and as our view of the latter may change with progressive knowledge, so may the conditions of correct thinking. At present, the Law of the Uniformity of Nature, including the Law of Causation, is regarded as the fundamental law to which our thinking must conform in order that our thought may agree with reality—in order that inferences drawn from data known to us by pure observation may agree with the facts of nature.

§ 17. From the Absolute Idealistic point of view as held by Hegel, Green and other British Neo-Hegelians, the fundamental Law of Thought is the law of thesis, antithesis and synthesis. Thought progresses by this law and produces the various things as it progresses. The things have no existence apart from thought They are thoughts abstracted from the thinking subject or self. The finite human self is a reproduction of the infinite or absolute self. The finite thought of man is a reflection of the absolute thought of God. The latter is implicit in the former and must be ultimately developed. All rational beings think alike and think according to the fundamental dialectical law stated above. Logic from this point of view is the science of Reason. It is identical with Metaphysics and Theology. It comprehends all the sciences of Nature and Spirit. The reason in man is identical with the Divine Reason. It develops and passes through stages or categories through which

the Divine Reason may be regarded as having passed in the act of creation. But creation is not an act in time; and the process is therefore a logical or dialectical one and not one in time. Logic is the science of the Categories of Thought. These categories are the chief stages through which Thought develops in man, in all rational Beings, starting from the notion of Being and attaining its highest form in Self-Consciousness. Reason is the system of the Categories of Thought. The law of Reason is the dialectical law of the development of thought and the chief stages are Being, Quality, Quantity, Degree, Identity, Ground, Substance, Cause, Reciprocity, Mechanism, Chemism, Teleology, Life, Cognition, Idea

§ 18. (3) Metaphysically, the inquiry is into the constituent elements of Thought.

Metaphysics attempts to analyse the unit of thought into its constituent elements. Kant's analysis is well-known. No thought, no cognition, no consciousness is possible, unless there be:—

(1) a conscious subject,

(2) an object thought of,

(3) a relation between the two.

There must be an object in time or in both time and space; and there must be a subject to be conscious of it, to perceive it or to think of it.

Kant gives an account of the elements contributed by the Subject to its thought as it is developed in the varying phases of perception, knowledge, idea or ideal, duty, beauty, end or purpose. According to him, space, time, the twelve categories, the ideas of the soul, the world and God, the categorical imperative, the æsthetic and teleological ideas are *a priori* elements contributed by the Subject to its thought at the various stages of its development.

The inquiry into the historical development of thought in the individual and in the race is regarded as *Psychological.*

The inquiry into the truth or falsity of thought or into the value of thought for knowledge is regarded as Logical.

The inquiry into the *a priori* elements of all thought is regarded as *Metaphysical.*

Each of these three different inquiries has its own Laws of Thought.

Printed at the Baptist Mission Press, Calcutta.